Obvious 13 *Co-separability is the dual of separability.*

Proposition 17 *Let $\mathfrak{A}$ be a filtrator. $\mathfrak{A}$ is a filtrator with co-separable core iff*

$$\forall x, y \in \mathfrak{A} : (x \equiv^{\mathfrak{A}} y \Rightarrow \exists X \in \operatorname{down} x, Y \in \operatorname{down} y : X \equiv^{\mathfrak{A}} Y).$$

Proof By duality. □

4.3. Intersecting and joining with an element of the core

Definition 49 *I call* ***down-aligned*** *filtrator such a filtrator $(\mathfrak{A}; \mathfrak{Z})$ that $\mathfrak{A}$ and $\mathfrak{Z}$ have common least element. (Let's denote it 0.)*

Definition 50 *I call* ***up-aligned*** *filtrator such a filtrator $(\mathfrak{A}; \mathfrak{Z})$ that $\mathfrak{A}$ and $\mathfrak{Z}$ have common greatest element. (Let's denote it 1.)*

Theorem 27 *For a filtrator $(\mathfrak{A}; \mathfrak{Z})$ where $\mathfrak{Z}$ is a boolean lattice, for every $B \in \mathfrak{Z}$, $\mathcal{A} \in \mathfrak{A}$:*

1. *$B \asymp^{\mathfrak{A}} \mathcal{A} \Leftrightarrow \overline{B} \supseteq \mathcal{A}$ if it is down-aligned, with finitely meet-closed and separable core;*
2. *$B \equiv^{\mathfrak{A}} \mathcal{A} \Leftrightarrow \overline{B} \subseteq \mathcal{A}$ if it is up-aligned, with finitely join-closed and co-separable core.*

Proof We will prove only the first as the second is dual.

$$\begin{aligned}
B \asymp^{\mathfrak{A}} \mathcal{A} &\Leftrightarrow \\
\exists A \in \operatorname{up} \mathcal{A} : B \asymp^{\mathfrak{A}} A &\Leftrightarrow \\
\exists A \in \operatorname{up} \mathcal{A} : B \cap^{\mathfrak{A}} A = 0 &\Leftrightarrow \\
\exists A \in \operatorname{up} \mathcal{A} : B \cap^{\mathfrak{Z}} A = 0 &\Leftrightarrow \\
\exists A \in \operatorname{up} \mathcal{A} : \overline{B} \supseteq A &\Leftrightarrow \\
\overline{B} \in \operatorname{up} \mathcal{A} &\Leftrightarrow \\
\overline{B} \supseteq \mathcal{A}.
\end{aligned}$$

□

5. Filters

5.1. Filters on posets

Let $\mathfrak{A}$ be a poset (partially ordered set) with the partial order $\subseteq$. I will call it **the base poset**.

Definition 51 ***Filter base*** *is a nonempty subset F of $\mathfrak{A}$ such that*

$$\forall X, Y \in F \exists Z \in F : (Z \subseteq X \wedge Z \subseteq Y).$$