Proof $\operatorname{Cor} a = \bigcap^{\mathfrak{Z}} \operatorname{up} a \subseteq \bigcap^{\mathfrak{A}} \operatorname{up} a = a$. □

Corollary 6 *$\operatorname{Cor} a \in \operatorname{down} a$ whenever $\operatorname{Cor} a$ exists for any element a of a filtered filtrator.*

Theorem 25 *$\operatorname{Cor}' a \subseteq a$ whenever $\operatorname{Cor}' a$ exists for any element a of a filtrator with join-closed core.*

Proof $\operatorname{Cor}' a = \bigcup^{\mathfrak{Z}} \operatorname{down} a = \bigcup^{\mathfrak{A}} \operatorname{down} a \subseteq a$. □

Corollary 7 *$\operatorname{Cor}' a \in \operatorname{down} a$ whenever $\operatorname{Cor}' a$ exists for any element a of a filtrator with join-closed core.*

Proposition 15 *$\operatorname{Cor}' a \subseteq \operatorname{Cor} a$ whenever both $\operatorname{Cor} a$ and $\operatorname{Cor}' a$ exist for any element a of a filtrator with join-closed core.*

Proof $\operatorname{Cor} a = \bigcap^{\mathfrak{Z}} \operatorname{up} a \supseteq \operatorname{Cor}' a$ because $\forall A \in \operatorname{up} a : \operatorname{Cor}' a \subseteq A$. □

Theorem 26 *$\operatorname{Cor}' a = \operatorname{Cor} a$ whenever both $\operatorname{Cor} a$ and $\operatorname{Cor}' a$ exist for any element a of a filtered filtrator.*

Proof It is with join-closed core because it is semifiltered. So $\operatorname{Cor}' a \subseteq \operatorname{Cor} a$. $\operatorname{Cor} a \in \operatorname{down} a$. So $\operatorname{Cor} a \subseteq \bigcup^{\mathfrak{Z}} \operatorname{down} a = \operatorname{Cor}' a$. □

Obvious 12 *$\operatorname{Cor}' a = \max \operatorname{down} a$ for an element a of a filtrator with join-closed core.*

4.2. Filtrators with separable core

Definition 47 *Let $\mathfrak{A}$ be a filtrator. $\mathfrak{A}$ is a **filtrator with separable core** when*

$$\forall x, y \in \mathfrak{A} : (x \asymp^{\mathfrak{A}} y \Rightarrow \exists X \in \operatorname{up} x : X \asymp^{\mathfrak{A}} y).$$

Proposition 16 *Let $\mathfrak{A}$ be a filtrator. $\mathfrak{A}$ is a filtrator with separable core iff*

$$\forall x, y \in \mathfrak{A} : (x \asymp^{\mathfrak{A}} y \Rightarrow \exists X \in \operatorname{up} x, Y \in \operatorname{up} y : X \asymp^{\mathfrak{A}} Y).$$

Proof

$\Rightarrow$ Apply the definition twice.

$\Leftarrow$ Obvious.

□

Definition 48 *Let $\mathfrak{A}$ be a filtrator. $\mathfrak{A}$ is a **filtrator with co-separable core** when*

$$\forall x, y \in \mathfrak{A} : (x \equiv^{\mathfrak{A}} y \Rightarrow \exists X \in \operatorname{down} x : X \equiv^{\mathfrak{A}} y).$$