(4)⇒(2) Let formula (4) holds. Then for any elements $a \subset b$ exists $c \neq 0$ such that $c \subseteq b$ and $c \cap a = 0$. Because $\mathfrak{A}$ is atomic there exists atom $d \subseteq c$. $d \in \operatorname{atoms} b$ and $d \notin \operatorname{atoms} a$. So $\operatorname{atoms} a \neq \operatorname{atoms} b$ and $\operatorname{atoms} a \subseteq \operatorname{atoms} b$. Consequently $\operatorname{atoms} a \subset \operatorname{atoms} b$.

□

4. Filtrators

Definition 31 *I will call a* ***filtrator*** *a pair* $(\mathfrak{A}; \mathfrak{Z})$ *of a poset* $\mathfrak{A}$ *and its subset* $\mathfrak{Z} \subseteq \mathfrak{A}$. *I call* $\mathfrak{A}$ *the* ***base*** *of a filtrator and* $\mathfrak{Z}$ *the* ***core*** *of a filtrator.*

Definition 32 *I will call a* ***lattice filtrator*** *a pair* $(\mathfrak{A}; \mathfrak{Z})$ *of a lattice* $\mathfrak{A}$ *and its subset* $\mathfrak{Z} \subseteq \mathfrak{A}$.

Definition 33 *I will call a* ***complete lattice filtrator*** *a pair* $(\mathfrak{A}; \mathfrak{Z})$ *of a complete lattice* $\mathfrak{A}$ *and its subset* $\mathfrak{Z} \subseteq \mathfrak{A}$.

Definition 34 *I will call a* ***central filtrator*** *a filtrator* $(\mathfrak{A}; Z(\mathfrak{A}))$ *where* $Z(\mathfrak{A})$ *is the center of a bounded lattice* $\mathfrak{A}$.

Remark 8 One use of filtrators is the theory of filters where the base lattice (or the lattice of principal filters) is essentially considered as the core of the lattice of filters. See below for a more exact formulation. Our primary interest is the properties of filters on sets (that is the filtrator of filters on a set), but instead we will research more general theory of filtrators.

Remark 9 An other important example of filtrators is **filtrator of funcoids** whose base is the set of funcoids [11] and whose core is the set of binary relations (or discrete funcoids).

Definition 35 *I will call* ***element*** *of a filtrator an element of its base.*

Definition 36 $\operatorname{up} a = \{c \in \mathfrak{Z} \mid c \supseteq a\}$ *where* $a \in \mathfrak{A}$.

Definition 37 $\operatorname{down} a = \{c \in \mathfrak{Z} \mid c \subseteq a\}$ *where* $a \in \mathfrak{A}$.

Obvious 8 "up" *and* "down" *are dual.*

The main purpose of this text is knowing properties of the core of a filtrator to infer properties of the base of the filtrator, specifically properties of $\operatorname{up} a$ for every element a.

Definition 38 *I call a filtrator with* ***join-closed core*** *such filtrator* $(\mathfrak{A}; \mathfrak{Z})$ *that* $\bigcup^{\mathfrak{Z}} S = \bigcup^{\mathfrak{A}} S$ *whenever* $\bigcup^{\mathfrak{Z}} S$ *exists for* $S \in \mathscr{P}\mathfrak{Z}$.