(1)⇒(3) Let $a, b \in \mathfrak{A}$. Let $fa = fb \Rightarrow a = b$. Let $a \subset b$. $fa \neq fb$ because $a \neq b$. $fa \subseteq fb$ because $a \subseteq b$. So $fa \subset fb$.

(2)⇒(1) Let $a, b \in \mathfrak{A}$. Let $fa \subseteq fb \Rightarrow a \subseteq b$. Let $fa = fb$. Then $a \subseteq b \wedge b \subseteq a$ and consequently $a = b$.

(3)⇒(2) Let $\forall a, b \in \mathfrak{A} : (a \subset b \Rightarrow fa \subset fb)$. Let $a \nsubseteq b$. Then $a \supset a \cap b$. So $fa \supset f(a \cap b)$. If $fa \subseteq fb$ then $fa \subseteq f(a \cap b)$ what is a contradiction.

(3)⇒(5)⇒(4) Obvious.

(4)⇒(3) Because $a \subset b \Rightarrow a \subseteq b \Rightarrow fa \subseteq fb$.

(5)⇔(6) Obvious.

□

3.2. Separation subsets and full stars

Definition 25 *$\partial_Y a = \{x \in Y \mid x \not\asymp a\}$ for an element a of a poset $\mathfrak{A}$ and $Y \in \mathcal{P}\mathfrak{A}$.*

Definition 26 ***Full star*** *of a is $\star a = \partial_{\mathfrak{A}} a$.*

Proposition 11 *If $\mathfrak{A}$ is a meet-semilattice, then $\star$ is a straight monotone map.*

Proof Monotonicity is obvious. Let $\star a \nsubseteq \star(a \cap b)$. Then it exists $x \in \star a$ such that $x \notin \star(a \cap b)$. So $x \cap a \notin \star b$ but $x \cap a \in \star a$ and consequently $\star a \nsubseteq \star b$. □

Definition 27 *A **separation subset** of a poset $\mathfrak{A}$ is such its subset Y that*

$$\forall a, b \in \mathfrak{A} : (\partial_Y a = \partial_Y b \Rightarrow a = b).$$

Definition 28 *I call **separable** such poset that $\star$ is an injection.*

Obvious 7 *A poset is separable iff it has separation subset.*

Definition 29 *A poset $\mathfrak{A}$ has **disjunction property of Wallman** iff for any $a, b \in \mathfrak{A}$ either $b \subseteq a$ or there exists a non-least element $c \subseteq b$ such that $a \asymp c$.*

Theorem 19 *For a meet-semilattice with least element the following statements are equivalent:*

1. *$\mathfrak{A}$ is separable.*

2. *$\forall a, b \in \mathfrak{A} : (\star a \subseteq \star b \Rightarrow a \subseteq b)$.*

3. *$\forall a, b \in \mathfrak{A} : (a \subset b \Rightarrow \star a \subset \star b)$.*

4. *$\forall a, b \in \mathfrak{A} : (a \subset b \Rightarrow \star a \neq \star b)$.*