4. $\forall a, b \in \mathfrak{A} : (fa \supset f(a \cap b) \Rightarrow fa \nsubseteq fb)$.

Proof

(1)⇔(2)⇔(3) Due $fa \supseteq f(a \cap b)$.

(3)⇔(4) Obvious.

□

Remark 7 The definition of straight map can be generalized for any poset $\mathfrak{A}$ by the formula

$$\forall a, b \in \mathfrak{A} : (fa \subseteq fb \Rightarrow \exists c \in \mathfrak{A} : (c \subseteq a \wedge c \subseteq b \wedge fa = fc)).$$

This generalization is not yet researched however.

Proposition 9 *Let f be a monotone map from a meet-semilattice $\mathfrak{A}$ to some poset $\mathfrak{B}$. If*

$$\forall a, b \in \mathfrak{A} : (f(a \cap b) = fa \cap fb)$$

then f is a straight map.

Proof Let $fa \subseteq fb$. Then $f(a \cap b) = fa \cap fb = fa$. □

Proposition 10 *Let f be a monotone map from a meet-semilattice $\mathfrak{A}$ to some poset $\mathfrak{B}$. If*

$$\forall a, b \in \mathfrak{A} : (fa \subseteq fb \Rightarrow a \subseteq b)$$

then f is a straight map.

Proof $fa \subseteq fb \Rightarrow a \subseteq b \Rightarrow a = a \cap b \Rightarrow fa = f(a \cap b)$. □

Theorem 18 *If f is a straight monotone map from a meet-semilattice $\mathfrak{A}$ then the following statements are equivalent:*

1. *f is an injection.*

2. $\forall a, b \in \mathfrak{A} : (fa \subseteq fb \Rightarrow a \subseteq b)$.

3. $\forall a, b \in \mathfrak{A} : (a \subset b \Rightarrow fa \subset fb)$.

4. $\forall a, b \in \mathfrak{A} : (a \subset b \Rightarrow fa \neq fb)$.

5. $\forall a, b \in \mathfrak{A} : (a \subset b \Rightarrow fa \not\supseteq fb)$.

6. $\forall a, b \in \mathfrak{A} : (fa \subseteq fb \Rightarrow a \not\supset b)$.

Proof