4. Obviously $(b \cup c) \setminus^* a \supseteq b \setminus^* a$ and $(b \cup c) \setminus^* a \supseteq c \setminus^* a$, thus $(b \cup c) \setminus^* a \supseteq (b \setminus^* a) \cup (c \setminus^* a)$. We have

$$\begin{aligned} (b \setminus^* a) \cup (c \setminus^* a) \cup a &= \\ ((b \setminus^* a) \cup a) \cup ((c \setminus^* a) \cup a) &= \\ (b \cup a) \cup (c \cup a) &= \\ a \cup b \cup c &\supseteq \\ b \cup c. \end{aligned}$$

From this by the definition of adjoints: $(b \setminus^* a) \cup (c \setminus^* a) \supseteq (b \cup c) \setminus^* a$.

□

Theorem 16 *$(\bigcup S) \setminus^* a = \bigcup \{ x \setminus^* a \mid x \in S \}$ for $a \in \mathfrak{A}$ and $S \in \mathscr{P}\mathfrak{A}$ where $\mathfrak{A}$ is a complete co-brouwerian lattice.*

Proof Because lower adjoint preserves all suprema. □

Theorem 17 *$(a \setminus^* b) \setminus^* c = a \setminus^* (b \cup c)$ for elements a, b, c of a complete co-brouwerian lattice.*

Proof $a \setminus^* b = \bigcap \{ z \in \mathfrak{A} \mid a \subseteq b \cup z \}$.
$(a \setminus^* b) \setminus^* c = \bigcap \{ z \in \mathfrak{A} \mid a \setminus^* b \subseteq c \cup z \}$.
$a \setminus^* (b \cup c) = \bigcap \{ z \in \mathfrak{A} \mid a \subseteq b \cup c \cup z \}$.
It's left to prove $a \setminus^* b \subseteq c \cup z \Leftrightarrow a \subseteq b \cup c \cup z$.
Let $a \setminus^* b \subseteq c \cup z$. Then $a \cup b \subseteq b \cup c \cup z$ by the lemma and consequently $a \subseteq b \cup c \cup z$.
Let $a \subseteq b \cup c \cup z$. Then $a \setminus^* b \subseteq (b \cup c \cup z) \setminus^* b \subseteq c \cup z$ by a theorem above. □

3. Straight maps and separation subsets

3.1. Straight maps

Definition 24 *Let f be a monotone map from a meet-semilattice $\mathfrak{A}$ to some poset $\mathfrak{B}$. I call f a* ***straight*** *map when*

$$\forall a, b \in \mathfrak{A} : (fa \subseteq fb \Rightarrow fa = f(a \cap b)).$$

Proposition 8 *The following statements are equivalent for a monotone map f:*

1. *f is a straight map.*
2. *$\forall a, b \in \mathfrak{A} : (fa \subseteq fb \Rightarrow fa \subseteq f(a \cap b))$.*
3. *$\forall a, b \in \mathfrak{A} : (fa \subseteq fb \Rightarrow fa \not\supset f(a \cap b))$.*