So $\langle f\rangle|_{\mathrm{up}\,\mathcal{F}\cap\mathscr{P}\mathrm{Base}(\mathcal{F})}=\langle f\rangle|_{\mathrm{up}\,\mathcal{F}}$ is a bijection from $\mathrm{up}\,\mathcal{F}\cap\mathscr{P}B$ to $\mathrm{up}(\uparrow^A\{a\}\times^{\mathsf{RLD}}\mathcal{F})\cap\mathscr{P}B$.

We have $\mathrm{up}\,\mathcal{F}\cap\mathscr{P}\mathrm{Base}(\mathcal{F})$ and $\mathrm{up}(\uparrow^A\{a\}\times^{\mathsf{RLD}}\mathcal{F})\cap\mathscr{P}B$ directly isomorphic and thus $\mathrm{up}\,\mathcal{F}$ is isomorphic to $\mathrm{up}(\uparrow^A\{a\}\times^{\mathsf{RLD}}\mathcal{F})$. □

Theorem 79. A monovalued reloid with atomic domain is atomic.

Proof. Let f is a monovalued reloid with atomic domain. There exists a function $F\in\mathrm{up}\,f$.

We have $f\subseteq\left(\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}F\right)|_{\mathrm{dom}\,f}$. Thus it suffices to prove that $\left(\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}F\right)|_{\mathrm{dom}\,f}$ is atomic.

Let the function $\tau\colon\mathrm{dom}\,F\to F$ is defined by the formula $\tau x=(x;Fx)$ (for every $x\in\mathrm{dom}\,F$).

That τ is an injection is obvious. That τ is a surjection is also obvious. Thus τ is a bijection.

Let $T=\langle\tau\rangle|_{\mathrm{up}\,\mathrm{dom}\,f\cap\mathscr{P}\mathrm{dom}\,F}$.

If $X\in\mathrm{up}\,\mathrm{dom}\,f\cap\mathscr{P}\mathrm{dom}\,F$ then

$$TX=\{\tau x\mid x\in X\}=\{(x;Fx)\mid x\in X\}=F|_X.$$

Thus $TX\subseteq F$ and $\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}TX\supseteq\left(\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}F\right)|_{\mathrm{dom}\,f}$. So

$$TX\in\mathrm{up}\left(\left(\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}F\right)|_{\mathrm{dom}\,f}\right)\cap\mathscr{P}F.$$

So $T\colon\mathrm{up}\,\mathrm{dom}\,f\cap\mathscr{P}\mathrm{dom}\,F\to\mathrm{up}\left(\left(\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}F\right)|_{\mathrm{dom}\,f}\right)\cap\mathscr{P}F$.

Let $X,Y\in\mathrm{up}\,\mathrm{dom}\,f\cap\mathscr{P}\mathrm{dom}\,F$ and $X\neq Y$. Then $TX=\langle\tau\rangle X\neq\langle\tau\rangle Y=TY$ because τ is a bijection. So T is an injection.

Let $Y\in\mathrm{up}\left(\left(\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}F\right)|_{\mathrm{dom}\,f}\right)\cap\mathscr{P}F$. Then $Y\subseteq F$ and thus $Y=F|_{\mathrm{dom}\,Y}$. We have $\mathrm{dom}\,Y\in\mathrm{up}\,\mathrm{dom}\,f\cap\mathscr{P}\mathrm{dom}\,F$ and

$$T\,\mathrm{dom}\,Y=\{\tau x\mid x\in\mathrm{dom}\,Y\}=\{(x;Fx)\mid x\in\mathrm{dom}\,Y\}=F|_{\mathrm{dom}\,Y}=Y.$$

Thus T is a surjection.

Thus T is a bijection and so $\mathrm{up}\;\mathrm{dom}\;f\;\cap\;\mathscr{P}\mathrm{dom}\;F$ is directly isomorphic to $\mathrm{up}\left(\left(\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}F\right)|_{\mathrm{dom}\,f}\right)\cap\mathscr{P}F$. Consequently $\mathrm{up}\;\mathrm{dom}\;f$ is isomorphic to $\mathrm{up}\left(\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}F\right)|_{\mathrm{dom}\,f}$, and so $\left(\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}F\right)|_{\mathrm{dom}\,f}$ is an atomic filter object because $\mathrm{dom}\,f$ is atomic by the assumption. □

Theorem 80. If f, g are reloids, $f\subseteq g$ and g is monovalued then $g|_{\mathrm{dom}\,f}=f$.

Proof. It's simple to show that $f=\bigcup\left\{f|_a\mid a\in\mathrm{atoms}\,1^{\mathfrak{F}(\mathrm{Src}\,f)}\right\}$ (use the fact that every atomic reloid $k\subseteq f|_a$ for some $a\in\mathrm{atoms}\,1^{\mathfrak{F}(\mathrm{Src}\,f)}$ and the fact that $\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)$ is atomistic).

Suppose that $g|_{\mathrm{dom}\,f}\neq f$. Then there exists $a\in\mathrm{atoms}\,\mathrm{dom}\,f$ such that $g|_a\neq f|_a$.

Obviously $g|_a\supseteq f|_a$.

If $g|_a\supset f|_a$ then $g|_a$ is not atomic (because $f|_a\neq 0^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}$) what contradicts to a theorem above. So $g|_a=f|_a$ what is a contradiction and thus $g|_{\mathrm{dom}\,f}=f$. □

Corollary 81. Every monovalued reloid is a restricted discrete monovalued reloid.

Proof. Let f is a monovalued reloid. Then exists a function $F\in\mathrm{up}\,f$. So we have

$$\left(\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}F\right)|_{\mathrm{dom}\,f}=f.$$ □

Corollary 82. Every monovalued injective reloid is a restricted injective monovalued discrete reloid.

Proof. Let f is a monovalued injective reloid. There exists a function F such that $f=\left(\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}F\right)|_{\mathrm{dom}\,f}$. Also there exists an injection $G\in\mathrm{up}\,f$.

Thus $f=f\cap\left(\left(\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}G\right)|_{\mathrm{dom}\,f}\right)=\left(\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}F\right)|_{\mathrm{dom}\,f}\cap\left(\left(\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}G\right)|_{\mathrm{dom}\,f}\right)=\left(\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}(F\cap G)\right)|_{\mathrm{dom}\,f}$. Obviously $F\cap G$ is an injection. □

Theorem 83. If a reloid f is monovalued and $\mathrm{dom}\,f$ is a principal f.o. then f is discrete.