Corollary 67. $I_{\mathcal{A}}^{\mathsf{RLD}}$ is isomorphic to $\mathcal{A}$ for every f.o. $\mathcal{A}$.

Theorem 68. There are atomic f.o. incomparable by Rudin-Keisler order.

Proof. See [2]. □

Theorem 69. $\geqslant_1$ and $\geqslant_2$ are different relations.

Proof. Consider a is an arbitrary non-empty f.o. Then $a \geqslant_1 0^{\mathfrak{F}(\mathrm{Base}(a))}$ but not $a \geqslant_2 0^{\mathfrak{F}(\mathrm{Base}(a))}$. □

Proposition 70. If $a \geqslant_2 b$ where a is an atomic f.o. then b is also an atomic f.o.

Proof. $b = \langle \uparrow f \rangle a$ for some $f : \mathrm{Base}(a) \rightarrow \mathrm{Base}(b)$. So b is an atomic f.o. since f is monovalued. □

Corollary 71. If $a \geqslant_1 b$ where a is an atomic f.o. then b is also an atomic f.o. or $0^{\mathfrak{F}(\mathrm{Base}(a))}$.

Proof. $b \subseteq \langle \uparrow f \rangle a$ for some $f : \mathrm{Base}(a) \rightarrow \mathrm{Base}(b)$. Therefore $b' = \langle \uparrow f \rangle a$ is an atomic f.o. From this follows our statement. □

Proposition 72. Principal filters, generated by sets of the same cardinality, are isomorphic.

Proof. Let A and B are sets of the same cardinality. Then there are a bijection f from A to B. We have $\langle f \rangle A = B$ and thus A and B are isomorphic. □

Proposition 73. If a filter object is isomorphic to a principal f.o., then it is also a principal f.o. induced by a set with the same cardinality.

Proof. Let A is a set and B is a f.o. isomorphic to A. Then there are sets $X \in \mathrm{up}\, A$ and $Y \in \mathrm{up}\, B$ such that there are a bijection $f : X \rightarrow Y$ such that $\langle f \rangle A = B$. Thus A is a set of the same cardinality as B. □

Proposition 74. A filter isomorphic to a non-trivial atomic f.o. is a non-trivial atomic f.o.

Proof. Let a is a non-trivial atomic f.o. and a is isomorphic to b. Then $a \geqslant_2 b$ and thus b is an atomic f.o. The f.o. b cannot be trivial because otherwise a would be also trivial. □

Theorem 75. For an infinite set U there exist $2^{2^{\mathrm{card U}}}$ equivalence classes of isomorphic ultrafilters.

Proof. The number of bijections between any two given subsets of U is no more than $(\mathrm{card}\, U)^{\mathrm{card}\, U} = 2^{\mathrm{card}\, U}$. The number of bijections between all pairs of subsets of U is no more than $2^{\mathrm{card}\, U} \cdot 2^{\mathrm{card}\, U} = 2^{\mathrm{card}\, U}$. Therefore each isomorphism class contains at most $2^{\mathrm{card}\, U}$ ultrafilters. But there are $2^{2^{\mathrm{card}\, U}}$ ultrafilters. So there are $2^{2^{\mathrm{card}\, U}}$ classes. □

Remark 76. One of the above mentioned equivalence classes contains trivial ultrafilters.

Corollary 77. There exist non-isomorphic nontrivial ultrafilters on any infinite set.

5 Consequences

Theorem 78. The reloid $\uparrow^A \{a\} \times^{\mathsf{RLD}} \mathcal{F}$ is isomorphic to the filter object $\mathcal{F}$ for every set A and $a \in A$.

Proof. Consider $B = \{a\} \times \mathrm{Base}(\mathcal{F})$ and $f = \{(x; (a; x)) \mid x \in \mathrm{Base}(\mathcal{F})\}$. Then f is a bijection from $\mathrm{Base}(\mathcal{F})$ to B.

If $X \in \mathrm{up}\, \mathcal{F}$ then $\langle f \rangle X \subseteq B$ and $\langle f \rangle X = \{a\} \times X \in \mathrm{up}(\uparrow^A \{a\} \times^{\mathsf{RLD}} \mathcal{F})$.

For every $Y \in \mathrm{up}(\uparrow^A \{a\} \times^{\mathsf{RLD}} \mathcal{F}) \cap \mathscr{P}B$ we have $Y = \{a\} \times X$ for some $X \in \mathrm{up}\, \mathcal{F}$ and thus $Y = \langle f \rangle X$.