Theorem 44. The following are categories (with reloid composition):

1. $\mathbf{MonRld}_{\subseteq,\supseteq}$;
2. $\mathbf{MonRld}_{\subseteq,=}$;
3. $\mathbf{MonRld}_{=,=}$.
4. $\mathbf{CoMonRld}_{\subseteq,\supseteq}$;
5. $\mathbf{CoMonRld}_{\subseteq,=}$;
6. $\mathbf{CoMonRld}_{=,=}$.

Proof. We will prove only the first three. The rest follow from duality. We need to prove only that composition of morphisms is a morphism, because associativity and existence of identity morphism are evident. We have:

1. Let $f \in \mathrm{Mor}_{\mathbf{MonRld}_{\subseteq,\supseteq}}(\mathcal{A};\mathcal{B})$, $g \in \mathrm{Mor}_{\mathbf{MonRld}_{\subseteq,\supseteq}}(\mathcal{B};\mathcal{C})$. Then $\operatorname{dom} f \subseteq \mathcal{A}$, $\operatorname{im} f \supseteq \mathcal{B}$, $\operatorname{dom} g \subseteq \mathcal{B}$, $\operatorname{im} g \supseteq \mathcal{C}$. So $\operatorname{dom}(g\circ f) \subseteq \mathcal{A}$, $\operatorname{im}(g\circ f) \supseteq \mathcal{C}$ that is $g\circ f \in \mathrm{Mor}_{\mathbf{MonRld}_{\subseteq,\supseteq}}(\mathcal{A};\mathcal{C})$.
2. Let $f \in \mathrm{Mor}_{\mathbf{MonRld}_{\subseteq,=}}(\mathcal{A};\mathcal{B})$, $g \in \mathrm{Mor}_{\mathbf{MonRld}_{\subseteq,=}}(\mathcal{B};\mathcal{C})$. Then $\operatorname{dom} f \subseteq \mathcal{A}$, $\operatorname{im} f = \mathcal{B}$, $\operatorname{dom} g \subseteq \mathcal{B}$, $\operatorname{im} g = \mathcal{C}$. So $\operatorname{dom}(g\circ f) \subseteq \mathcal{A}$, $\operatorname{im}(g\circ f) = \mathcal{C}$ that is $g\circ f \in \mathrm{Mor}_{\mathbf{MonRld}_{\subseteq,=}}(\mathcal{A};\mathcal{C})$.
3. Let $f \in \mathrm{Mor}_{\mathbf{MonRld}_{=,=}}(\mathcal{A};\mathcal{B})$, $g \in \mathrm{Mor}_{\mathbf{MonRld}_{=,=}}(\mathcal{B};\mathcal{C})$. Then $\operatorname{dom} f = \mathcal{A}$, $\operatorname{im} f = \mathcal{B}$, $\operatorname{dom} g = \mathcal{B}$, $\operatorname{im} g = \mathcal{C}$. So $\operatorname{dom}(g\circ f) = \mathcal{A}$, $\operatorname{im}(g\circ f) = \mathcal{C}$ that is $g\circ f \in \mathrm{Mor}_{\mathbf{MonRld}_{=,=}}(\mathcal{A};\mathcal{C})$. □

Definition 45. Let **BijRld** is the groupoid of all bijections of the category of reloid triples. Its objects are filter objects and its morphisms from a f.o. $\mathcal{A}$ to f.o. $\mathcal{B}$ are monovalued injective reloids f such that $\operatorname{dom} f = \mathcal{A}$ and $\operatorname{im} f = \mathcal{B}$.

Theorem 46. Filter objects $\mathcal{A}$ and $\mathcal{B}$ are isomorphic iff $\mathrm{Mor}_{\mathbf{BijRld}}(\mathcal{A};\mathcal{B}) \neq \emptyset$.

Proof.

⇒. Let $\mathcal{A}$ and $\mathcal{B}$ are isomorphic. Then there are sets $A \in \operatorname{up}\mathcal{A}$, $B \in \operatorname{up}\mathcal{B}$ and a bijective **Set**-morphism $F: A \to B$ such that $\langle F\rangle: \mathscr{P}A \cap \operatorname{up}\mathcal{A} \to \mathscr{P}B \cap \operatorname{up}\mathcal{B}$ is a bijection.

Obviously $f = (\uparrow^{\mathsf{RLD}}F)|_{\mathcal{A}}$ is monovalued and injective.

$\operatorname{im} f = \bigcap \{\uparrow^B \operatorname{im} G \mid G \in \operatorname{up}(\uparrow^{\mathsf{RLD}}F)|_{\mathcal{A}}\} = \bigcap \{\uparrow^B \operatorname{im}(H \cap F|_X) \mid H \in \operatorname{up}(\uparrow^{\mathsf{RLD}}F)|_{\mathcal{A}}, X \in \operatorname{up}\mathcal{A}\} = \bigcap \{\uparrow^B \operatorname{im} F|_P \mid P \in \operatorname{up}\mathcal{A}\} = \bigcap \{\uparrow^B \langle F\rangle P \mid P \in \operatorname{up}\mathcal{A}\} = \bigcap \{\uparrow^B \langle F\rangle P \mid P \in \mathscr{P}A \cap \operatorname{up}\mathcal{A}\} = \bigcap \langle\uparrow^B\rangle(\mathscr{P}B \cap \operatorname{up}\mathcal{B}) = \bigcap \langle\uparrow^B\rangle \operatorname{up}\mathcal{B} = \mathcal{B}$.

Thus $\operatorname{dom} f = \mathcal{A}$ and $\operatorname{im} f = \mathcal{B}$.

⇐. Let f is a monovalued injective reloid such that $\operatorname{dom} f = \mathcal{A}$ and $\operatorname{im} f = \mathcal{B}$. Then exist a function F' and an injective binary relation F'' such that $F', F'' \in \operatorname{up} f$. Thus $F = F' \cap F''$ is an injection such that $F \in \operatorname{up} f$. The function F is a bijection from $A = \operatorname{dom} F$ to $B = \operatorname{im} F$. The function $\langle F\rangle$ is an injection on $\mathscr{P}A \cap \operatorname{up}\mathcal{A}$ (and moreover on $\mathscr{P}A$). It's simple to show that $\forall X \in \mathscr{P}A \cap \operatorname{up}\mathcal{A}: \langle F\rangle X \in \mathscr{P}B \cap \operatorname{up}\mathcal{B}$ and similarly $\forall Y \in \mathscr{P}B \cap \operatorname{up}\mathcal{B}$: $\langle F\rangle^{-1} Y = \langle F^{-1}\rangle Y \in \mathscr{P}A \cap \operatorname{up}\mathcal{A}$. Thus $\langle F\rangle|_{\mathscr{P}A \cap \operatorname{up}\mathcal{A}}$ is a bijection $\mathscr{P}A \cap \operatorname{up}\mathcal{A} \to \mathscr{P}B \cap \operatorname{up}\mathcal{B}$. So filter objects $\mathcal{A}$ and $\mathcal{B}$ are isomorphic. □

Proposition 47. $(\geqslant_1) = (\subseteq) \circ (\geqslant_2)$ (when we limit to small f.o.).

Proof. $\mathcal{A} \geqslant_1 \mathcal{B}$ iff exists a function $f: \operatorname{Base}(\mathcal{A}) \to \operatorname{Base}(\mathcal{B})$ such that $\mathcal{B} \subseteq \langle\uparrow f\rangle\mathcal{A}$. But $\mathcal{B} \subseteq \langle\uparrow f\rangle\mathcal{A}$ is equivalent to $\exists \mathcal{B}' \in \mathfrak{F}: (\mathcal{B} \subseteq \mathcal{B}' \wedge \mathcal{B}' = \langle\uparrow f\rangle\mathcal{A})$. So $\mathcal{A} \geqslant_1 \mathcal{B}$ is equivalent to existence of $\mathcal{B}' \in \mathfrak{F}$ such that $\mathcal{B} \subseteq \mathcal{B}'$ and existence of a function $f: \operatorname{Base}(\mathcal{A}) \to \operatorname{Base}(\mathcal{B})$ such that $\mathcal{B}' = \langle\uparrow f\rangle\mathcal{A}$. That is equivalent to $\mathcal{A}((\subseteq) \circ (\geqslant_2))\mathcal{B}$. □

Proposition 48. If a and b is an atomic f.o. then $b \geqslant_1 a \Leftrightarrow b \geqslant_2 a$.

Proof. We need to prove only $b \geqslant_1 a \Rightarrow b \geqslant_2 a$. If $b \geqslant_1 a$ then there exists a monovalued reloid $f: \operatorname{Base}(b) \to 1^{\mathfrak{F}(\operatorname{Base}(a))}$ such that $\operatorname{dom} f = b$ and $\operatorname{im} f \supseteq a$. Then $\operatorname{im} f = \operatorname{im}(\mathsf{FCD})f \in \{0^{\mathfrak{F}(\operatorname{Base}(a))}\} \cup \operatorname{atoms} 1^{\mathfrak{F}(\operatorname{Base}(a))}$ because $(\mathsf{FCD})f$ is a monovalued funcoid. So $\operatorname{im} f = a$ (taken in account $a \neq 0^{\mathfrak{F}(\operatorname{Base}(a))}$) and thus $b \geqslant_2 a$. □