Orderings of filters in terms of reloids*

Extensions of Rudin-Keisler ordering

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August 13, 2013

Abstract

Orderings of filters which extend Rudin-Keisler preorder of ultrafilters are described in terms of reloids that (roughly speaking) is filters on sets of binary relations between some sets. Also it is defined isomorphism of filters which extends Rudin-Keisler equivalence of ultrafilters.

Keywords: Rudin-Keisler order, Rudin-Keisler preorder, Rudin-Keisler ordering, filters, ultrafilters, reloids, isomorphism, isomorphic

A.M.S. subject classification: 06F99, 06A99, 54D99

1 Draft status

It is a draft.

2 Preliminary definitions

Whilist my other works use filters to research funcoids and reloids [3], here it is discussed the opposite thing, the theory of reloids is used to describe properties of filters.

See [4] for the definition of filter objects and [3] for the definition and properties of reloids and funcoids.

I will call *small* sets members of some Grothendieck universe.

Recall that morphisms of the category **Set** (or "**Set**-morphisms" for short) are triples $(F; A; B)$ of a function F and small sets A and B where $\operatorname{dom} F \subseteq A$ and $\operatorname{im} F \subseteq B$.

For $X \in \mathscr{P}A$ we'll denote $\langle (F; A; B) \rangle X = \langle F \rangle X$.

Let $f = (F; A; B)$ is a **Set**-morphism. I will denote in this article

$$\uparrow f = \left(\uparrow^{\mathsf{FCD}(A;B)} F; A; B\right) \quad \text{and} \quad \uparrow^{\mathsf{RLD}} f = \left(\uparrow^{\mathsf{RLD}(A;B)} F; A; B\right).$$

2.1 Equivalent filters

We will restrict to small sets that is members of some Grothendieck universe.

Definition 1. Two filter objects $\mathcal{A}$ and $\mathcal{B}$ (with possibly different base sets) are *equivalent* ($\mathcal{A} \sim \mathcal{B}$) iff there exists a set X such that $X \in \operatorname{up} \mathcal{A}$ and $X \in \operatorname{up} \mathcal{B}$ and $\mathscr{P}X \cap \operatorname{up} \mathcal{A} = \mathscr{P}X \cap \operatorname{up} \mathcal{B}$.

Proposition 2. If two filter objects with the same base are equivalent they are equal.

Proof. Let $\mathcal{A}$ and $\mathcal{B}$ are two f.o. and $\mathscr{P}X \cap \operatorname{up} \mathcal{A} = \mathscr{P}X \cap \operatorname{up} \mathcal{B}$ for some set X such that $X \in \operatorname{up} \mathcal{A}$ and $X \in \operatorname{up} \mathcal{B}$, and $\operatorname{Base}(\mathcal{A}) = \operatorname{Base}(\mathcal{B})$. Then $\operatorname{up} \mathcal{A} = (\mathscr{P}X \cap \operatorname{up} \mathcal{A}) \cup \{ Y \in \mathscr{P}\operatorname{Base}(\mathcal{A}) \mid Y \supseteq X \} = (\mathscr{P}X \cap \operatorname{up} \mathcal{B}) \cup \{ Y \in \mathscr{P}\operatorname{Base}(\mathcal{B}) \mid Y \supseteq X \} = \operatorname{up} \mathcal{B}$. □

Proposition 3. $\sim$ restricted to small filter objects is an equivalence relation.

Proof.

Reflexivity. Obvious.

Symmetry. Obvious.

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