Reloids are a special case of trans-reloids (take $A=B$).

In order to not repeat the entire theory of reloids for the more general case of trans-reloids, I will describe properties of trans-reloids through reloids equivalent (in the sense I defined above for filter objects) to them.

Definition 11. I will call trans-reloids f and g *composable* when $\operatorname{Dst} f=\operatorname{Src} g$.

Definition 12. Composition of trans-reloids is defined by the formula

$$g\circ f=\bigcap^{\mathsf{RLD}(\operatorname{Src} f;\operatorname{Dst} g)}\{G\circ F\mid F\in\operatorname{up} f,G\in\operatorname{up} g\}.$$

Composition of trans-reloids is a trans-reloid.

Theorem 13. Let f, g, f', g' are trans-reloids. Then $f\sim f'\wedge g\sim g'\Rightarrow g\circ f\sim g'\circ f'$.

Proof. There exist a binary relation K such that $K\in\operatorname{up} f$, $K\in\operatorname{up} f'$, and $\mathscr{P}K\cap\operatorname{up} f=\mathscr{P}K\cap\operatorname{up} f'$ and a binary relation L such that $L\in\operatorname{up} g$, $L\in\operatorname{up} g'$, and $\mathscr{P}L\cap\operatorname{up} g=\mathscr{P}L\cap\operatorname{up} g'$. $g\circ f=\bigcap^{\mathsf{RLD}(\operatorname{Src} f;\operatorname{Dst} g)}\{G\circ F\mid F\in\operatorname{up} f,\ G\in\operatorname{up} g\}=\bigcap^{\mathsf{RLD}(\operatorname{Src} f;\operatorname{Dst} g)}\{G\circ F\mid F\in\mathscr{P}K\cap\operatorname{up} f,\ G\in\mathscr{P}K\cap\operatorname{up} g\}=\bigcap^{\mathsf{RLD}(\operatorname{Src} f;\operatorname{Dst} g)}\{G\circ F\mid F\in\mathscr{P}K\cap\operatorname{up} f',\ G\in\mathscr{P}K\cap\operatorname{up} g'\}\sim\bigcap^{\mathsf{RLD}(\operatorname{Src} f';\operatorname{Dst} g')}\{G\circ F\mid F\in\mathscr{P}K\cap\operatorname{up} f',G\in\mathscr{P}K\cap\operatorname{up} g'\}=\bigcap^{\mathsf{RLD}(\operatorname{Src} f';\operatorname{Dst} g')}\{G\circ F\mid F\in\operatorname{up} f',\ G\in\operatorname{up} g'\}=g'\circ f'$. ☐

Theorem 14. The following are equivalent for every three composable trans-reloids f, g, h:

1. $h=g\circ f$.
2. There exist some set and reloids f' and g' on this set such that $f'\sim f$, $g'\sim g$, and $h\sim g'\circ f'$.
3. For every reloids f' and g' such that $f'\sim f$, $g'\sim g$ we have $h\sim g'\circ f'$.

Proof.

(1) ⇒ (2). Let $h=g\circ f$. Get $f'=f\div(A\cup B\cup C)^2$ and $g'=g\div(A\cup B\cup C)^2$. Obviously $f'\sim f$, $g'\sim g$. $g'\circ f'=\bigcap^{\mathsf{RLD}(A\cup B\cup C)}\{G\circ F\mid F\in\operatorname{up} f',G\in\operatorname{up} g'\}=\bigcap^{\mathsf{RLD}(A\cup B\cup C)}\{G\circ F\mid F\in\operatorname{up} f,\ G\in\operatorname{up} g\}\sim\bigcap^{\mathsf{RLD}(A;C)}\{G\circ F\mid F\in\operatorname{up} f,G\in\operatorname{up} g\}=g\circ f$.

(2) ⇒ (3). From the previous theorem.

(3) ⇒ (1). Let $f'=f$ and $g'=g$. We have $h\sim g\circ f$ and thus $h=g\circ f$. ☐

Theorem 15. Composition of trans-reloids is associative.

Proof. Let $f\colon A\rightarrow B$, $g\colon B\rightarrow C$, $h\colon C\rightarrow D$ are trans-reloids. Consider the reloids

$$f'=f\div(A\cup B\cup C\cup D)^2,\quad g'=g\div(A\cup B\cup C\cup D)^2,\quad h'=h\div(A\cup B\cup C\cup D)^2.$$

Then $g'\circ f'\sim g\circ f$ and $h'\circ g'\sim h\circ g$ thus $h\circ(g\circ f)\sim h'\circ(g'\circ f')=(h'\circ g')\circ f'\sim(h\circ g)\circ f$.

Thus $h\circ(g\circ f)=(h\circ g)\circ f$. ☐

It is simple to show that trans-reloids with the given composition and identity functions as identities form a category. I'll call it *the category of trans-reloids*.

2.2.1 Some definitions for trans-reloids

Definition 16. The *reverse* trans-reloid of a trans-reloid f is defined by the formula

$$\operatorname{up} f^{-1}=\left\{F^{-1}\mid F\in\operatorname{up} f^{-1}\right\}.$$

Definition 17. *Domain* and *image* of a trans-reloid f are defined as follows:

$$\operatorname{dom} f=\bigcap^{\mathfrak{F}(\operatorname{Dst} f)}\langle\operatorname{dom}\rangle\operatorname{up} f;\quad \operatorname{im} f=\bigcap^{\mathfrak{F}(\operatorname{Dst} f)}\langle\operatorname{im}\rangle\operatorname{up} f.$$