If $z \subseteq a \wedge z \cap^{\mathfrak{F}} b = 0^{\mathfrak{A}}$ then $z' = \bigcup \left\{ x \in \operatorname{atoms} z \mid x \cap^{\mathfrak{A}} b = 0^{\mathfrak{A}} \right\}$ is defined. z' is a lower bound for $\left\{ z \in \operatorname{atoms} a \mid z \cap^{\mathfrak{A}} b = 0^{\mathfrak{A}} \right\}$.

Thus $z' \in \left\{ z \in \mathfrak{A} \mid z \subseteq a \wedge z \cap^{\mathfrak{A}} b = 0^{\mathfrak{A}} \right\}$ and so $\bigcup \left\{ z \in \operatorname{atoms} a \mid z \cap^{\mathfrak{A}} b = 0^{\mathfrak{A}} \right\}$ is an upper bound of $\left\{ z \in \mathfrak{A} \mid z \subseteq a \wedge z \cap^{\mathfrak{A}} b = 0^{\mathfrak{A}} \right\}$.

If y is above every $z' \in \left\{ z \in \mathfrak{A} \mid z \subseteq a \wedge z \cap^{\mathfrak{A}} b = 0^{\mathfrak{A}} \right\}$ then y is above every $z \in \operatorname{atoms} a$ such that $z \cap^{\mathfrak{A}} b = 0^{\mathfrak{A}}$ and thus y is above $\bigcup \left\{ z \in \operatorname{atoms} a \mid z \cap^{\mathfrak{A}} b = 0^{\mathfrak{A}} \right\}$.

Thus $\bigcup \left\{ z \in \operatorname{atoms} a \mid z \cap^{\mathfrak{A}} b = 0^{\mathfrak{A}} \right\}$ is least upper bound of

$$\left\{ z \in \mathfrak{A} \mid z \subseteq a \wedge z \cap^{\mathfrak{A}} b = 0^{\mathfrak{A}} \right\},$$

that is

$$\bigcup \left\{ z \in \mathfrak{A} \mid z \subseteq a \wedge z \cap^{\mathfrak{A}} b = 0^{\mathfrak{A}} \right\} = \bigcup \left\{ z \in \operatorname{atoms} a \mid z \cap^{\mathfrak{A}} b = 0^{\mathfrak{A}} \right\} = \bigcup (\operatorname{atoms} a \setminus \operatorname{atoms} b).$$

□

Note that $\mathfrak{F}$ is co-brouwerian by corollary 11 in [1] and atomistic by theorem 48 in [1], so our theorem applies to the lattice $\mathfrak{F}$, and more generally to any filters on a boolean lattice.

Proposition 1 *For filters on boolean lattices the three above ways to express quasidifference of a and b are also equal to $\bigcup^{\mathfrak{F}} \{ a \cap^{\mathfrak{F}} \uparrow \overline{B} \mid B \in \operatorname{up} b \}$ ($\uparrow X$ denotes the principal filter induced by X).*

Remark 1 *By corollary 8 in [1] the set of filters on a boolean lattice is complete. So our formula is well-defined.*

Proof Using results from [1]:

$\bigcup^{\mathfrak{F}} \{ z \in \mathfrak{F} \mid z \subseteq a \wedge z \cap b = 0^{\mathfrak{F}} \} \subseteq \bigcup^{\mathfrak{F}} \{ a \cap^{\mathfrak{F}} \uparrow \overline{B} \mid B \in \operatorname{up} b \}$ because

$$\begin{gathered} z \in \{ z \in \mathfrak{F} \mid z \subseteq a \wedge z \cap b = 0^{\mathfrak{F}} \} \Leftrightarrow z \subseteq a \wedge z \cap b = 0^{\mathfrak{F}} \Leftrightarrow \\ z \subseteq a \wedge \exists B \in \operatorname{up} b : z \cap \uparrow B = 0^{\mathfrak{F}} \Leftrightarrow z \subseteq a \wedge \exists B \in \operatorname{up} b : z \subseteq \uparrow \overline{B} \Leftrightarrow \\ \exists B \in \operatorname{up} b : (z \subseteq a \wedge z \subseteq \uparrow \overline{B}) \Leftrightarrow \exists B \in \operatorname{up} b : z \subseteq a \cap^{\mathfrak{F}} \uparrow \overline{B} \Rightarrow \\ z \subseteq \bigcup^{\mathfrak{F}} \{ a \cap^{\mathfrak{F}} \uparrow \overline{B} \mid B \in \operatorname{up} b \}. \end{gathered}$$

But obviously $a \cap^{\mathfrak{F}} \uparrow \overline{B} \in \{ z \in \mathfrak{F} \mid z \subseteq a \wedge z \cap b = 0^{\mathfrak{F}} \}$ and thus

$$a \cap^{\mathfrak{F}} \uparrow \overline{B} \subseteq \bigcup^{\mathfrak{F}} \{ z \in \mathfrak{F} \mid z \subseteq a \wedge z \cap b = 0^{\mathfrak{F}} \}$$

and so $\bigcup^{\mathfrak{F}} \{ z \in \mathfrak{F} \mid z \subseteq a \wedge z \cap b = 0^{\mathfrak{F}} \} \supseteq \bigcup^{\mathfrak{F}} \{ a \cap^{\mathfrak{F}} \uparrow \overline{B} \mid B \in \operatorname{up} b \}$. □

The above proposition completes the proof of problem 1 in [1].