Proposition 32. $(\mathcal{X} \div A) \div B = \mathcal{X} \div B$ if $B \subseteq A$.

Proof. $(\mathcal{X} \div A) \div B = \bigsqcap \{\uparrow^B (Y \cap B) \mid Y \in \bigsqcap \{\uparrow^A (X \cap A) \mid X \in \mathcal{X}\}\} = \bigsqcap \{\uparrow^B (X \cap A) \mid X \in \mathcal{X}\} \sqcap \uparrow^B B = \bigsqcap \{\uparrow^B (X \cap A \cap B) \mid X \in \mathcal{X}\} = \mathcal{X} \div (A \cap B) = \mathcal{X} \div B$. □

5.2 Category Rel

Category **Rel** with the identity up-arrow functor to itself and "reverse relation" as the dagger is an obvious example of a partially ordered dagger category under **Rel**.

Definition 33. $f \div (A \times B) = (A; B; (\mathrm{GR}\, f) \div (A \times B))$ for every **Rel**-morphism f.

Proposition 34. $\iota_{A,B} f = (A; B; \mathrm{GR}\, f \cap (A \times B))$.

Proof. $\iota_{A,B} f = (\mathrm{Dst}\, f \rightleftarrows B) \circ f \circ (A \rightleftarrows \mathrm{Src}\, f) = (A; B; \mathrm{GR}\, f \cap (A \times B))$. □

5.3 Category FCD

Category FCD with the up-arrow functor $\uparrow^{\mathsf{FCD}}$ and "reverse funcoid" as the dagger is a partially ordered dagger category under **Rel**.

Proposition 35. $A \rightleftarrows^{\mathsf{FCD}} B = (A; B; \lambda \mathcal{X} \in \mathfrak{F}(A): \mathcal{X} \div B; \lambda \mathcal{Y} \in \mathfrak{F}(B): \mathcal{Y} \div A)$ for objects $A \subseteq B$ of FCD.

Proof. $\langle A \rightleftarrows^{\mathsf{FCD}} B \rangle \mathcal{X} = \bigsqcap \{\langle A \rightleftarrows^{\mathsf{FCD}} B \rangle^* X \mid X \in \mathcal{X}\} = \bigsqcap \{\uparrow^B \langle A \rightleftarrows B \rangle X \mid X \in \mathcal{X}\} = \bigsqcap \{\uparrow^B (X \cap A \cap B) \mid X \in \mathcal{X}\} = \bigsqcap \{\uparrow^B (X \cap B) \mid X \in \mathcal{X}\} = \mathcal{X} \div B$.

Rest follows from symmetry. □

Proposition 36.

1. $\langle A \rightleftarrows^{\mathsf{FCD}} B \rangle^* X = \uparrow^B X$ for every $X \in \mathscr{P} A$ if $A \subseteq B$.
2. $\langle (B \rightleftarrows^{\mathsf{FCD}} A) \rangle^* Y = \uparrow^A (Y \cap A)$ for every $Y \in \mathscr{P} B$ if $A \subseteq B$.

Proof. By definition of principal funcoid. □

5.4 Category RLD

Category RLD with the up-arrow functor $\uparrow^{\mathsf{RLD}}$ and "reverse reloid" as the dagger is a partially ordered dagger category under **Rel**.

Obvious 37. $A \rightleftarrows^{\mathsf{RLD}} B = \uparrow^{\mathsf{RLD}(A;B)} \mathrm{id}_{A \cap B}$.

Definition 38. $f \div (A \times B) = (A; B; (\mathrm{GR}\, f) \div (A \times B))$ for every reloid f.

Proposition 39. $\iota_{A,B} f = f \div (A \times B)$.

Proof. $\iota_{A,B} f = (\mathrm{Dst}\, f \rightleftarrows^{\mathsf{RLD}} B) \circ f \circ (A \rightleftarrows^{\mathsf{RLD}} \mathrm{Src}\, f) = \bigsqcap \{\uparrow^{\mathsf{RLD}} ((\mathrm{Dst}\, f \rightleftarrows B) \circ F \circ (A \rightleftarrows \mathrm{Src}\, f)) \mid F \in \mathrm{xyGR}\, f\} = \bigsqcap \{\uparrow^{\mathsf{RLD}} (F \cap (A \times B)) \mid F \in \mathrm{xyGR}\, f\} = f \div (A \times B)$. [TODO: Filters on cartesian products vs reloids.] □

6 Equalizers

Categories $\mathbf{cont}(\mathcal{C})$ are defined in [1].

I will denote W the forgetful functor from $\mathbf{cont}(\mathcal{C})$ to $\mathcal{C}$.

In the definition of the category $\mathbf{cont}(\mathcal{C})$ take values of $\uparrow$ as principal morphisms. [TODO: Wording.]