$A \sqcap B \in \overline{F} \Leftrightarrow A \in \overline{F} \vee B \in \overline{F}$ is equivalent to $\neg(A \sqcap B \in F) \Leftrightarrow \neg(A \in F \wedge B \in F)$ is equivalent to $A \sqcap B \in F \Leftrightarrow A \in F \wedge B \in F$.

We have the following commutative diagram in category Set, every arrow of this diagram is an isomorphism, every cycle in this diagram is an identity:

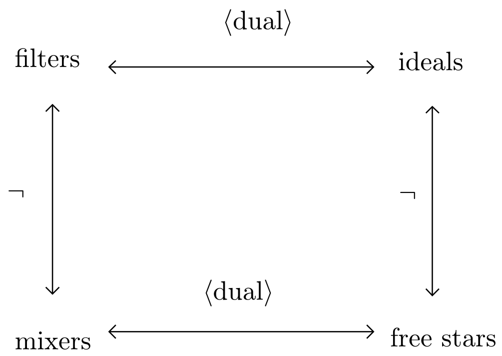

Figure 1. Diagram Υ

(where $\neg$ denotes set-theoretic complement).

These isomorphisms are also order isomorphisms if we define order in the right way.

The above it is defined for lattices only. Generalizing this for arbitrary posets is straigthforward:

Definition 2. Let $\mathfrak{A}$ be a poset.

- *Filters* are sets F without the greatest element of $\mathfrak{A}$ with $A, B \in F \Leftrightarrow \exists Z \in F: (Z \sqsubseteq A \wedge Z \sqsubseteq B)$ (for every $A, B \in \mathfrak{Z}$).
- *Ideals* are sets F without the least element of $\mathfrak{A}$ with $A, B \in F \Leftrightarrow \exists Z \in F: (Z \sqsupseteq A \wedge Z \sqsupseteq B)$ (for every $A, B \in \mathfrak{Z}$).
- *Free stars* are sets F without the greatest element of $\mathfrak{A}$ with

  $A, B \in \overline{F} \Leftrightarrow \exists Z \in \overline{F}: (Z \sqsupseteq A \wedge Z \sqsupseteq B)$

  $A \notin F \wedge B \notin F \Leftrightarrow \exists Z \in \overline{F}: (Z \sqsupseteq A \wedge Z \sqsupseteq B)$

  $A \in F \vee B \in F \Leftrightarrow \neg \exists Z \in \overline{F}: (Z \sqsupseteq A \wedge Z \sqsupseteq B)$
- *Mixers* are lower sets F without the least element of $\mathfrak{A}$ with $\neg \exists Z \in \overline{F}: (Z \sqsubseteq A \wedge Z \sqsubseteq B) \Leftrightarrow A \in F \vee B \in F$ or equivalently $\exists Z \in \overline{F}: (Z \sqsubseteq A \wedge Z \sqsubseteq B) \Leftrightarrow A \notin F \wedge B \notin F$ (for every $A, B \in \mathfrak{Z}$).

Proposition 3. The following are equivelent: [TODO: With one side implications and requirement to be upper/lower set.]

1. F is a free star.
2. $\forall Z \in \mathfrak{A}: (Z \sqsupseteq A \wedge Z \sqsupseteq B \Rightarrow Z \in F) \Leftrightarrow A \in F \vee B \in F$ for every $A, B \in \mathfrak{A}$ and $F \neq \mathscr{P}\mathfrak{A}$.

Proof. The following is a chain of equivalencies:

$$\exists Z \in \overline{F}: (Z \sqsupseteq A \wedge Z \sqsupseteq B) \Leftrightarrow A \notin F \wedge B \notin F;$$
$$\forall Z \in \overline{F}: \neg(Z \sqsupseteq A \wedge Z \sqsupseteq B) \Leftrightarrow A \in F \vee B \in F;$$
$$\forall Z \in \mathfrak{A}: (Z \notin F \Rightarrow \neg(Z \sqsupseteq A \wedge Z \sqsupseteq B)) \Leftrightarrow A \in F \vee B \in F;$$
$$\forall Z \in \mathfrak{A}: (Z \sqsupseteq A \wedge Z \sqsupseteq B \Rightarrow Z \in F) \Leftrightarrow A \in F \vee B \in F.$$

□