It's enough to prove

$$\bigsqcap \left\{ \uparrow^{\mathsf{RLD}(\operatorname{Src} f \times \operatorname{Dst} g; \mho)} (G \otimes f) \mid G \in \operatorname{GR} \bigsqcup T \right\} = \bigsqcup \left\{ \bigsqcap \left\langle \uparrow^{\mathsf{RLD}(\operatorname{Src} f \times \operatorname{Dst} g; \mho)} \right\rangle (G \otimes F) \mid G \in T \right\}$$

but this is the statement of the lemma. □

4 Embedding reloids into funcoids

Definition 8. Let f is a reloid. The funcoid $\rho f \in \mathsf{FCD}(\operatorname{Src} f; \operatorname{Dst} f)$ is defined by the formulas:

$$\langle \rho f \rangle x = f \circ x \quad \text{and} \quad \langle \rho f^{-1} \rangle y = f^{-1} \circ y$$

where x are endo-reloids on $\operatorname{Src} f$ and y are endo-reloids on $\operatorname{Dst} f$.

Proposition 9. It is really a funcoid (if we equate reloids x and y with corresponding filters on cartesian products of sets).

Proof. $y \not\asymp \langle \rho f \rangle x \Leftrightarrow y \not\asymp f \circ x \Leftrightarrow f^{-1} \circ y \not\asymp x \Leftrightarrow \langle \rho f^{-1} \rangle y \not\asymp x$. □

Corollary 10. $(\rho f)^{-1} = \rho f^{-1}$.

Definition 11. It can be continued to arbitrary funcoids x having source $\operatorname{Src} f$ by the formula $\langle \rho^* f \rangle x = \langle \rho f \rangle \mathrm{id}_{\operatorname{Src} f} \circ x$.

Proposition 12. ρ is an injection.

Proof. Consider $x = \mathrm{id}_{\operatorname{Src} f}$. □

Proposition 13. $\rho(g \circ f) = (\rho g) \circ (\rho f)$.

Proof. $\langle \rho(g \circ f) \rangle x = g \circ f \circ x = \langle \rho g \rangle \langle \rho f \rangle x = (\langle \rho g \rangle \circ \langle \rho f \rangle) x$. Thus $\langle \rho(g \circ f) \rangle = \langle \rho g \rangle \circ \langle \rho f \rangle = \langle (\rho g) \circ (\rho f) \rangle$ and so $\rho(g \circ f) = (\rho g) \circ (\rho f)$. □

Theorem 14. $\rho \bigsqcup F = \bigsqcup \langle \rho \rangle F$ for a set F of reloids.

Proof. It's enough to prove $\langle \rho \bigsqcup F \rangle^* X = \langle \bigsqcup \langle \rho \rangle F \rangle^* X$ for a set X.

Really, $\langle \rho \bigsqcup F \rangle^* X = \langle \rho \bigsqcup F \rangle \uparrow X = \bigsqcup F \circ \uparrow X = \bigsqcup \{ f \circ \uparrow X \mid f \in F \} = \bigsqcup \{ \langle \rho f \rangle \uparrow X \mid f \in F \} = \langle \bigsqcup \{ \rho f \mid f \in F \} \rangle \uparrow X = \langle \bigsqcup \langle \rho \rangle F \rangle^* X$. □

Conjecture 15. $\rho \bigsqcap F = \bigsqcap \langle \rho \rangle F$ for a set F of reloids.

Proposition 16. $\rho \mathrm{id}^{\mathsf{RLD}(A)} = \mathrm{id}^{\mathsf{FCD}(A)}$.

Proof. $\left\langle \rho \mathrm{id}^{\mathsf{RLD}(A)} \right\rangle x = \mathrm{id}^{\mathsf{RLD}(A)} \circ x = x$. □

We can try to develop further theory by applying embedding of reloids into funcoids for researching of properties of reloids.

Theorem 17. Reloid f is monovalued iff funcoid ρf is monovalued.

Proof. ρf is monovalued$\Leftrightarrow (\rho f) \circ (\rho f)^{-1} \sqsubseteq 1_{\operatorname{Dst} \rho f} \Leftrightarrow \rho(f \circ f^{-1}) \sqsubseteq 1_{\operatorname{Dst} f} \Leftrightarrow \rho(f \circ f^{-1}) \sqsubseteq \mathrm{id}^{\mathsf{RLD}(A)} \Leftrightarrow \rho(f \circ f^{-1}) \sqsubseteq \rho \mathrm{id}^{\mathsf{FCD}(A)} \Leftrightarrow f \circ f^{-1} \sqsubseteq \mathrm{id}^{\mathsf{FCD}(A)} \Leftrightarrow f$ is monovalued. □

Bibliography

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