Let $X\in\operatorname{dom}\bigsqcap\left\{\uparrow^{\mathsf{RLD}(\operatorname{Src} f\times\operatorname{Dst} g;\mho)}(G\otimes F)\mid F\in\operatorname{GR} f,G\in\operatorname{GR} g\right\}$. Then there exist Y such that $X\times Y\in\bigsqcap\left\{\uparrow^{\mathsf{RLD}(\operatorname{Src} f\times\operatorname{Dst} g;\mho)}(G\otimes F)\mid F\in\operatorname{GR} f,G\in\operatorname{GR} g\right\}$. So because it is a generalized filter base $X\times Y\supseteq G\otimes F$ for some $F\in\operatorname{GR} f$, $G\in\operatorname{GR} g$. Thus $X\in\operatorname{dom}(G\otimes F)$, $X\sqsupseteq\uparrow^{\mathsf{RLD}(\operatorname{Src} f;\operatorname{Dst} g)}\operatorname{dom}(G\otimes F)$, $X\in\operatorname{GR}(g\circ f)$. □

We can define $g\otimes f$ for reloids f, g:

$$g\otimes f=\{G\otimes F\mid F\in\operatorname{GR} f,G\in\operatorname{GR} g\}.$$

Then

$$g\circ f=\bigsqcap\left\langle\uparrow^{\mathsf{RLD}(\operatorname{Src} f;\operatorname{Dst} g)}\right\rangle\langle\operatorname{dom}\rangle(g\otimes f)=\operatorname{dom}\bigsqcap\left\langle\uparrow^{\mathsf{RLD}(\operatorname{Src} f\times\operatorname{Dst} g;\mho)}\right\rangle(g\otimes f).$$

2 Lemmas for the main result

Let $F=\uparrow^{\mathsf{RLD}(\operatorname{Src} F;\operatorname{Dst} F)}f$ is a principal reloid.

Lemma 5. $(g\otimes f)\cup(h\otimes f)=(g\cup h)\otimes f$ for binary relations f, g, h.

Proof. $(g\cup h)\otimes f=\Theta_0\,f\cap\Theta_1\,(g\cup h)=\Theta_0\,f\cap(\Theta_1\,g\cup\Theta_1\,h)=(\Theta_0\,f\cap\Theta_1\,g)\cup(\Theta_0\,f\cap\Theta_1\,h)=(g\otimes f)\cup(h\otimes f)$. □

Lemma 6. $\bigsqcap\left\{\uparrow^{\mathsf{RLD}(\operatorname{Src} f\times\operatorname{Dst} g;\mho)}(G\otimes f)\mid G\in\bigsqcup T\right\}=\bigsqcup\left\{\bigsqcap\left\langle\uparrow^{\mathsf{RLD}(\operatorname{Src} f\times\operatorname{Dst} g;\mho)}\right\rangle(G\otimes F)\mid G\in T\right\}$.

Proof. $\bigsqcap\left\{\uparrow^{\mathsf{RLD}(\operatorname{Src} f\times\operatorname{Dst} g;\mho)}(G\otimes f)\mid G\in\operatorname{GR}\bigsqcup T\right\}\sqsupseteq\bigsqcup\left\{\bigsqcap\left\langle\uparrow^{\mathsf{RLD}(\operatorname{Src} f\times\operatorname{Dst} g;\mho)}\right\rangle(G\otimes F)\mid G\in T\right\}$ is obvious.

Let $K\in\bigsqcup\left\{\bigsqcap\left\langle\uparrow^{\mathsf{RLD}(\operatorname{Src} f\times\operatorname{Dst} g;\mho)}\right\rangle(G\otimes F)\mid G\in T\right\}$. Then for each $G\in T$

$$K\in\bigsqcap\left\langle\uparrow^{\mathsf{RLD}(\operatorname{Src} f\times\operatorname{Dst} g;\mho)}\right\rangle(G\otimes F);$$

$K\in\bigsqcap\left\langle\uparrow^{\mathsf{RLD}(\operatorname{Src} f\times\operatorname{Dst} g;\mho)}\right\rangle\{\Gamma\otimes f\mid\Gamma\in\operatorname{GR} G\}$.

$K\in\{(\Gamma_0\otimes f)\cap\ldots\cap(\Gamma_n\otimes f)\mid n\in\mathbb{N},\Gamma_i\in\operatorname{GR} G\}$.

$\forall G\in T\colon K\supseteq(\Gamma_{G,0}\otimes f)\cap\ldots\cap(\Gamma_{G,n}\otimes f)$ for some $n\in\mathbb{N},\Gamma_{G,i}\in G$.

Let $G\in\bigcap T$.

$K\supseteq(\Gamma_0\otimes f)\cap\ldots\cap(\Gamma_n\otimes f)$ where $\Gamma_i=\bigcup_{g\in G}\Gamma_{g,i}\in\operatorname{GR}\bigsqcup T$.

$K\in\{(\Gamma_0\otimes f)\cap\ldots\cap(\Gamma_n\otimes f)\mid n\in\mathbb{N}\}$.

So $K\in\{(\Gamma'_0\otimes f)\cap\ldots\cap(\Gamma'_n\otimes f)\mid n\in\mathbb{N},\Gamma'_i\in\operatorname{GR}\bigsqcup T\}=\bigsqcap\left\langle\uparrow^{\mathsf{RLD}(\operatorname{Src} f\times\operatorname{Dst} g;\mho)}\right\rangle\{G\otimes f\mid G\in\operatorname{GR}\bigsqcup T\}$. □

3 Proof of the main result

Theorem 7. $(\bigsqcup T)\circ F=\bigsqcup\{G\circ F\mid G\in T\}$ for every principal reloid $F=\uparrow^{\mathsf{RLD}(\operatorname{Src} f;\operatorname{Dst} g)}f$.

Proof.

$$\begin{aligned}\left(\bigsqcup T\right)\circ F &= \bigsqcap\left\langle\uparrow^{\mathsf{RLD}(\operatorname{Src} f;\operatorname{Dst} g)}\right\rangle\langle\operatorname{dom}\rangle\left(\left(\bigsqcup T\right)\otimes F\right)\\ &= \operatorname{dom}\bigsqcap\left\langle\uparrow^{\mathsf{RLD}(\operatorname{Src} f\times\operatorname{Dst} g;\mho)}\right\rangle\left(\left(\bigsqcup T\right)\otimes F\right)\\ &= \operatorname{dom}\bigsqcap\left\langle\uparrow^{\mathsf{RLD}(\operatorname{Src} f\times\operatorname{Dst} g;\mho)}\right\rangle\left\{G\otimes f\mid G\in\operatorname{GR}\bigsqcup T\right\}\end{aligned}$$

$$\begin{aligned}\bigsqcup\{G\circ F\mid G\in T\} &= \bigsqcup\left\{\bigsqcap\left\langle\uparrow^{\mathsf{RLD}(\operatorname{Src} f;\operatorname{Dst} g)}\right\rangle\langle\operatorname{dom}\rangle(G\otimes F)\mid G\in T\right\}\\ &= \bigsqcup\left\{\operatorname{dom}\bigsqcap\left\langle\uparrow^{\mathsf{RLD}(\operatorname{Src} f\times\operatorname{Dst} g;\mho)}\right\rangle(G\otimes F)\mid G\in T\right\}\\ &= \operatorname{dom}\bigsqcup\left\{\bigsqcap\left\langle\uparrow^{\mathsf{RLD}(\operatorname{Src} f\times\operatorname{Dst} g;\mho)}\right\rangle(G\otimes F)\mid G\in T\right\}.\end{aligned}$$