

This article presents another version of star-composition of funcoids.

Let a be an anchored relation of the form \mathfrak{A} and $\text{dom } \mathfrak{A} = n$.

Let every f_i (for all $i \in n$) be a pointfree funcoid with $\text{Src } f_i = \mathfrak{A}_i$.

The star-composition of a with f is an anchored relation of the form $\lambda i \in \text{dom } \mathfrak{A}$: $\text{Dst } f_i$ defined by the formula [TODO: it should have a distinct notation with the other star-composition of staroids]

$$L \in \text{GR StarComp}(a; f) \Leftrightarrow (\lambda i \in n: \langle f_i^{-1} \rangle L_i) \in \text{GR } a.$$

Theorem 1. Let $\text{Src } f_i$ be separable join-semilattice and $\text{Dst } f_i$ be a starrish join-semilattice for every $i \in n$ for a set n . Let $\text{form } a = \prod_{i \in n} (\text{Src } f_i)$

1. If a is a prestaroid then $\text{StarComp}(a; f)$ is a prestaroid.
2. If a is a staroid then $\text{StarComp}(a; f)$ is a staroid.
3. If a is a complementary staroid and then $\text{StarComp}(a; f)$ is a complementary staroid.

Proof. We have $\langle f_i^{-1} \rangle (X \sqcup Y) = \langle f_i^{-1} \rangle X \sqcup \langle f_i^{-1} \rangle Y$.

1. Let $L \in \prod_{i \in (\text{arity } f) \setminus \{k\}} (\text{form } f_i)$ for some $k \in n$ and $X, Y \in \text{form } f_k$. Then

$$\begin{aligned} X \sqcup Y \in \langle \text{StarComp}(a; f) \rangle_k L &\Leftrightarrow \\ (\lambda i \in \text{dom } f: \langle f_i^{-1} \rangle \left(\begin{array}{c} X \sqcup Y \text{ if } i = k \\ L_i \text{ if } i \neq k \end{array} \right)_i) \in \text{GR } a &\Leftrightarrow \\ (\lambda i \in \text{dom } f: \left(\begin{array}{c} \langle f_i^{-1} \rangle X \sqcup \langle f_i^{-1} \rangle Y \text{ if } i = k \\ \langle f_i^{-1} \rangle L_i \text{ if } i \neq k \end{array} \right)_i) \in \text{GR } a &\Leftrightarrow \\ \langle f_i^{-1} \rangle X \sqcup \langle f_i^{-1} \rangle Y \in \langle a \rangle_k (\lambda i \in (\text{dom } f) \setminus \{k\}: \langle f_i^{-1} \rangle L_i) &\Leftrightarrow \\ \langle f_i^{-1} \rangle X \in \langle a \rangle_k (\lambda i \in n \setminus \{k\}: \langle f_i^{-1} \rangle L_i) \vee \langle f_i^{-1} \rangle Y \in \langle a \rangle_k (\lambda i \in n \setminus \{k\}: \langle f_i^{-1} \rangle L_i) &\Leftrightarrow \\ \left(\lambda i \in \text{dom } f: \left(\begin{array}{c} \langle f_i^{-1} \rangle X \text{ if } i = k \\ \langle f_i^{-1} \rangle L_i \text{ if } i \neq k \end{array} \right)_i \right) \in \text{GR } a \vee \left(\lambda i \in \text{dom } f: \right. & \\ \left. \left(\begin{array}{c} \langle f_i^{-1} \rangle Y \text{ if } i = k \\ \langle f_i^{-1} \rangle L_i \text{ if } i \neq k \end{array} \right)_i \right) \in \text{GR } a &\Leftrightarrow \\ \left(\lambda i \in \text{dom } f: \langle f_i^{-1} \rangle \left(\begin{array}{c} X \text{ if } i = k \\ L_i \text{ if } i \neq k \end{array} \right)_i \right) \in \text{GR } a \vee \left(\lambda i \in \text{dom } f: \right. & \\ \left. \langle f_i^{-1} \rangle \left(\begin{array}{c} Y \text{ if } i = k \\ L_i \text{ if } i \neq k \end{array} \right)_i \right) \in \text{GR } a &\Leftrightarrow \\ X \in \langle \text{StarComp}(a; f) \rangle_k L \vee Y \in \langle \text{StarComp}(a; f) \rangle_k L. & \end{aligned}$$

Thus $\text{StarComp}(a; f)$ is a pre-staroid.

2. $\langle f_i \rangle$ are monotone functions by the proposition 15.14. Thus $\langle f_i^{-1} \rangle X_i \sqsubseteq \langle f_i^{-1} \rangle Y_i$ if $X, Y \in \prod_{i \in (\text{arity } f) \setminus \{k\}} (\text{form } f_i)$ and $X \sqsubseteq Y$. So if a is a staroid and $X \in \text{GR StarComp}(a; f)$ then $(\lambda i \in \text{dom } f: \langle f_i^{-1} \rangle X_i) \in \text{GR } a$ then $(\lambda i \in \text{dom } f: \langle f_i^{-1} \rangle Y_i) \in \text{GR } a$ that is $Y \in \text{GR StarComp}(a; f)$.

- 3.

$$\begin{aligned} L_0 \sqcup L_1 \in \text{GR StarComp}(a; f) &\Leftrightarrow \\ (\lambda i \in n: \langle f_i^{-1} \rangle (L_0 \sqcup L_1) i) \in \text{GR } a &\Leftrightarrow \\ (\lambda i \in n: \langle f_i^{-1} \rangle L_0 i \sqcup \langle f_i^{-1} \rangle L_1 i) \in \text{GR } a &\Leftrightarrow \\ \exists c \in \{0, 1\}: (\lambda i \in n: \langle f_i^{-1} \rangle L_{c(i)} i) \in \text{GR } a &\Leftrightarrow \\ \exists c \in \{0, 1\}: (\lambda i \in n: L_{c(i)} i) \in \text{GR StarComp}(a; f). & \end{aligned}$$

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