Proof First prove that $\{A \cap B \mid A \in a, B \in b\}$ is a filter. Let $X, Y \in \{A \cap B \mid A \in a, B \in b\}$. Then $X = A_1 \cap B_1$ and $Y = A_2 \cap B_2$ where $A_1, A_2 \in a$ and $B_1, B_2 \in b$. Consequently $X \cap Y = (A_1 \cap A_2) \cap (B_1 \cap B_2)$ where $A_1 \cap A_2 \in a$, $B_1 \cap B_2 \in b$; thus $X \cap Y \in \{A \cap B \mid A \in a, B \in b\}$. Let $X \in \{A \cap B \mid A \in a, B \in b\}$ and $C \supseteq X$. We have $X = A \cap B$ where $A \in a$, $B \in b$. We have $C = C \cup X = C \cup (A \cap B) = (C \cup A) \cap (C \cup B)$ where $C \cup A \in a$ and $C \cup B \in b$; thus $C \in \{A \cap B \mid A \in a, B \in b\}$. So $\{A \cap B \mid A \in a, B \in b\}$ is a filter.

We need to prove that $\{A \cap B \mid A \in a, B \in b\}$ is the lowest upper bound of $\{a, b\}$. We have $\{A \cap B \mid A \in a, B \in b\} \supseteq a$ because if $X \in a$ then $X = X \cap U \in \{A \cap B \mid A \in a, B \in b\}$. Similarly $\{A \cap B \mid A \in a, B \in b\} \supseteq b$. Thus it is an upper bound.

Let p is an upper bound of $\{a, b\}$. Then $p \supseteq a$ that is $\forall A \in a : A \in p$ and $\forall B \in b : B \in p$. Thus because p is a filter we have $\forall A \in a, B \in b : A \cap B \in p$ that is $p \supseteq \{A \cap B \mid A \in a, B \in b\}$. □

Proposition 6 $[A) \cup^{\mathcal{F}} [B) = [A \cap B)$ *for every subsets A and B of U.*

Proof We need to prove that $[A \cap B)$ is the least upper bound of $\{[A), [B)\}$.

That $[A \cap B) \supseteq [A), [B)$ is obvious.

Remained to prove that $\forall a \in \mathcal{F} : (a \supseteq [A), [B) \Rightarrow a \supseteq [A \cap B))$. Really,

$$a \supseteq [A), [B) \Rightarrow A, B \in a \Rightarrow A \cap B \in a \Rightarrow a \supseteq [A \cap B).$$

□

4.6.2. Uniform triples

I will define uniform connectedness. Below I will show that my definition is equivalent to the classical definition of uniform connectedness.

I will call a **uniform triple** on a set U the triple $(f; A; B)$ where f is a filter on $\mathscr{P}(U \times U)$ and A, B are such sets that $A \times B \in f$. Note that uniform spaces can be considered as uniform triples with $A = B$. I will denote $\mathcal{R}$ the set of filters on $\mathscr{P}(U \times U)$ and $\mathcal{U}$ the set of uniform triples.

I will call a **generalized uniform space** a uniform triple with $A = B$.

Remark 4 In fact there can be defined composition of uniform triples and they thus form morphisms of certain category. But in this article I'll not dive into details here. See my draft article [5].

We will introduce order on the set of uniform triples on a set by the formula

$$(f; A_0; B_0) \subseteq (g; A_1; B_1) \Leftrightarrow f \subseteq g \wedge A_0 \supseteq A_1 \wedge B_0 \supseteq B_1.$$

Easy to see that $(f; A_0; B_0) \cup^{\mathcal{U}} (g; A_1; B_1) = (f \cup^{\mathcal{R}} g; A_0 \cap A_1; B_0 \cap B_1)$.

For a morphism $(f; A; B)$ of the category of binary relations, I will denote $[(f; A; B)) = ([f); A; B)$. Easy to see that $[(f; A; B))$ is a uniform triple.

By abuse of notation I will denote

$$(f; A_0; B_0) \in (g; A_1; B_1) \Leftrightarrow f \in g \wedge A_0 = A_1 \wedge B_0 = B_1$$

where f is a binary relation and g is a filter on $\mathscr{P}(U \times U)$.