To get path connectedness we take (for some topology $\mathfrak{A}$)

$$\tau_A(x,y) \Leftrightarrow \exists f \in \mathrm{C}([0;1];\mathfrak{A}|_A) : (f(0) = x \wedge f(1) = y). \tag{3}$$

Definition 11 *We can define two connector spaces $T(U;\tau)$ and $Q(U;\tau)$ with the base U corresponding to a link space $(U;\tau)$ by the formulas:*

$$\forall X, Y \in \mathscr{P}U : (X\ T(U;\tau)\ Y \Leftrightarrow \forall x \in X, y \in Y : \tau(x,y,X \cup Y));$$
$$\forall X, Y \in \mathscr{P}U : (X\ Q(U;\tau)\ Y \Leftrightarrow \exists x \in X, y \in Y : \tau(x,y,X \cup Y)).$$

Obvious 4 *If τ is reflexive then $Q(U;\tau)$ is a normalized connector.*

Obvious 5

1. $(T(U;\tau))|_K = T((U;\tau)|_K)$;
2. $(Q(U;\tau))|_K = Q((U;\tau)|_K)$.

Proposition 1 $\mathrm{LC}(U;\tau) = \mathrm{CC}(T(U;\tau))$ *for every reflexive link space $(U;\tau)$.*

Proof Let A is connected regarding $T(U;\tau)$. Then

$$\forall X, Y \in \mathscr{P}A \setminus \{\emptyset\} : (X \cup Y = A \wedge X \cap Y = \emptyset \Rightarrow X\ T(U;\tau)\ Y)$$

that is

$$\forall X, Y \in \mathscr{P}A \setminus \{\emptyset\} : (X \cup Y = A \wedge X \cap Y = \emptyset \Rightarrow \forall x \in X, y \in Y : \tau(x,y,X \cup Y)).$$

Let $a, b \in A$ and $a \neq b$. Then exist $X, Y \in \mathscr{P}A \setminus \{\emptyset\}$ such that $X \cup Y = A \wedge X \cap Y = \emptyset$ and $a \in X$, $b \in Y$. So $\tau(a,b,X \cup Y)$ that is $\tau(a,b,A)$. So taking in account reflexivity of τ we get that A is connected regarding τ.

Let now A is connected regarding $(U;\tau)$. Let $X, Y \in \mathscr{P}A \setminus \{\emptyset\} \wedge X \cup Y = A \wedge X \cap Y = \emptyset$. We have $\tau(a,b,A)$ for every $a \in X$, $b \in Y$. Thus $X\ T(U;\tau)\ Y$. So A is connected regarding $T(U;\tau)$. □

Theorem 2 *For every equivalence link space $(U;\tau)$*

$$\mathrm{LC}(U;\tau) = \mathrm{CC}(T(U;\tau)) = \mathrm{CC}(Q(U;\tau)).$$

Proof Enough to prove $\mathrm{LC}(U;\tau) = \mathrm{CC}(Q(U;\tau))$.

Let A is not connected regarding $(U;\tau)$ that is there are $a, b \in A$ such that $\neg(a\ \tau_A\ b)$. Then $a \in K$ and $b \in A \setminus K$ where K is a equivalence class regarding τ_A. So $\neg(K\ Q(U;\tau)\ A \setminus K)$ and thus A is not connected regarding $Q(U;\tau)$.

Let A is connected regarding $(U;\tau)$. Then for every $X, Y \in \mathscr{P}A \setminus \{\emptyset\}$ we have $\forall x \in X, y \in Y : x\ \tau_A\ y$ and thus $\exists x \in X, y \in Y : x\ \tau_A\ y$ that is $X\ Q(U;\tau)\ Y$. So A is connected regarding $Q(U;\tau)$. □