4. Examples of connectedness

4.1. Topological connectedness

Let $\mathfrak{A}$ is a topological space. If we take

$$X \; r \; Y \Leftrightarrow \text{(X is not open or Y is not open)}$$

or

$$X \; r \; Y \Leftrightarrow \text{(X is not closed or Y is not closed)}$$

or

$$X \; r \; Y \Leftrightarrow \operatorname{cl}_{X\cup Y}(X) \cap Y \neq \emptyset \vee \operatorname{cl}_{X\cup Y}(Y) \cap X \neq \emptyset \qquad (2)$$

where openness and closedness is taken on the space $\mathfrak{A}$ restricted to the set $X \cup Y$ and cl_A means the closure on the subspace A, then we get the classical definition of a set connected regarding a topology.

Observe that there are several connectors which define the same connectedness (because their normalized connectors are identical).

4.2. Path connectedness and similar

Definition 6 *I will call a ternary relation $\tau \in \mathscr{P}(U \times U \times \mathscr{P}U)$* ***link****.*

I will call the pair $(U;\tau)$ a ***link space****.*

I will denote $a \; \tau_A \; b = \tau_A(a,b) = \tau(a,b,A)$.

Remark 1 The expression $\tau(a,b,A)$ generalizes the statement "There exists a path from a to b through A." (where path may be taken in the sense used in topology or the sense used in graph theory).

Definition 7 *I will call a link space $(U;\tau)$* ***increasing*** *iff*

$$\forall A, B \in \mathscr{P}U : (A \supseteq B \Rightarrow \tau_A \supseteq \tau_B).$$

Definition 8 *I will call the* ***restriction*** *of a link space$(U;\tau)$ to a set $A \in \mathscr{P}U$ the link space $(A;\tau \cap (A \times A \times \mathscr{P}A))$.*

Definition 9 *I call a link space $(U;\tau)$* ***symmetric*** *when τ_A is symmetric for every $A \in \mathscr{P}U$,* ***transitive*** *when τ_A is transitive for every $A \in \mathscr{P}U$,* ***reflexive*** *when τ_A is reflexive on A for every $A \in \mathscr{P}U$. I will call a link space* ***equivalence*** *when it is symmetric, transitive, and reflexive.*

Definition 10 *I will call a set A* ***connected regarding a link*** *τ when $\forall x, y \in A : \tau(x,y,A)$. I call* ***connectedness*** *regarding a link space $(U;\tau)$ the collection of all connected (regarding τ) sets on U. I will denote $\operatorname{LC}(U;\tau)$ the connectedness regarding $(U;\tau)$. ("LC" is deciphered as "link connectedness".)*