

Then  $g = \langle f \times f \rangle^* \Delta$ .

**Proof.** From the above  $\langle f \times f \rangle^* \Delta \sqsubseteq g \circ g^{-1} \sqsubseteq g$ . [FIXME: Funcooids and reloids are confused.]

It's remained to prove  $g \sqsubseteq \langle f \times f \rangle^* \Delta$ .

[FIXME: Possible errors.]

Suppose there is  $U \in \text{xyGR } \langle f \times f \rangle^* \Delta$  such that  $U \notin \text{GR } g$ .

Then  $\{V \setminus U \mid V \in \text{GR } g\} = g \setminus U$  would be a proper filter.

Thus by reflexivity  $\langle f \times f \rangle^*(g \setminus U) \neq 0$ .

By compactness of  $f \times f$ ,  $\text{Cor } \langle f \times f \rangle^*(g \setminus U) \neq 0$ .

Suppose  $\uparrow\{(x; x)\} \sqsubseteq \langle f \times f \rangle^*(g \setminus U)$ ; then  $g \setminus U \not\sqsubseteq \langle f^{-1} \times f^{-1} \rangle\{(x; x)\}$ ;  $U \sqsubseteq \langle f^{-1} \times f^{-1} \rangle\{(x; x)\} \sqsubseteq \langle f^{-1} \times f^{-1} \rangle \Delta$  what is impossible.

Thus there exist  $x \neq y$  such that  $\{(x; y)\} \sqsubseteq \text{Cor } \langle f \times f \rangle^*(g \setminus U)$ . Thus  $\{(x; y)\} \sqsubseteq \langle f \times f \rangle^* g$ .

Thus by the lemma  $\{(x; y)\} \sqsubseteq \Delta$  what is impossible. So  $U \in \text{GR } g$ .

We have  $\text{xyGR } \langle f \times f \rangle^* \Delta \sqsubseteq \text{GR } g$ ;  $\langle f \times f \rangle^* \Delta \sqsupseteq g$ . □

**Corollary 18.** Let  $f$  is a  $T_1$ -separable (the same as  $T_2$  for symmetric transitive) compact funcooid and  $g$  is a uniform space (reflexive, symmetric, and transitive endoreloid) such that  $(\text{FCD})g = f$ . Then  $g = \langle f \times f \rangle^* \Delta$ .

An (incomplete) attempt to prove one more theorem follows:

**Theorem 19.** Let  $\mu$  and  $\nu$  be uniform spaces,  $(\text{FCD})\mu$  be a compact funcooid. Then a map  $f$  is a continuous map from  $(\text{FCD})\mu$  to  $(\text{FCD})\nu$  iff  $f$  is a (uniformly) continuous map from  $\mu$  to  $\nu$ .

**Proof.** [FIXME: errors in this proof.]

We have  $\mu = \langle (\text{FCD})\mu \times (\text{FCD})\mu \rangle^{\uparrow \text{RLD}} \Delta$

$f \in C_?((\text{FCD})\mu; (\text{FCD})\nu)$ . Then

$$f \times f \in C_?((\text{FCD})(\mu \times \mu); (\text{FCD})(\nu \times \nu))$$

$$(f \times f) \circ (\text{FCD})(\mu \times \mu) \sqsubseteq (\text{FCD})(\nu \times \nu) \circ (f \times f)$$

For every  $V \in \text{GR}(\nu \times \nu)$  we have  $\langle g^{-1} \rangle V \in \langle (\text{FCD})(\mu \times \mu) \rangle \{y\}$  for some  $y$ .

$$\langle g^{-1} \rangle V \in \langle (\text{FCD})\mu \times (\text{FCD})\mu \rangle^{\uparrow \text{RLD}} \Delta = \text{GR } \mu$$

$$\langle g \rangle \langle g^{-1} \rangle V \sqsubseteq V$$

We need to prove  $f \in C(\mu; \nu)$  that is  $\forall p \in \text{GR } \nu \exists q \in \text{GR } \mu: \langle f \rangle q \sqsubseteq p$ . But this follows from the above. □

## Bibliography

- [1] Victor Porton. Categorical product of funcooids. At <http://www.mathematics21.org/binaries/product.pdf>.
- [2] Victor Porton. *Algebraic General Topology. Volume 1*. 2013.