7. $(\mathsf{RLD})_{\mathrm{in}}(f^{-1}\circ f)$ [TODO: Use this below.]

Proof. ?? □

Proposition 10. Let g be a reloid and $f=(\mathsf{FCD})g$. Then $\langle f\times f\rangle^{*}\Delta\sqsupseteq g$.

Proof. $\langle f\times f\rangle^{*}\Delta\not\asymp\uparrow^{\mathsf{RLD}}Y\Leftrightarrow\uparrow^{\mathsf{RLD}}\Delta\,[f\times f]\uparrow^{\mathsf{RLD}}Y\Leftrightarrow\uparrow^{\mathsf{FCD}}\Delta\left[f\times^{(C)}f\right]\uparrow^{\mathsf{FCD}}Y\Leftrightarrow f\circ\uparrow^{\mathsf{FCD}}\Delta\circ f^{-1}\not\asymp\uparrow^{\mathsf{FCD}}Y\Leftrightarrow f\circ f^{-1}\not\asymp\uparrow^{\mathsf{FCD}}Y\Leftrightarrow f\not\asymp\uparrow^{\mathsf{FCD}}Y\Leftrightarrow f\sqcap\uparrow^{\mathsf{FCD}}Y\neq 0\Leftarrow(\mathsf{RLD})_{\mathrm{in}}(f\sqcap\uparrow^{\mathsf{FCD}}Y)\neq 0\Leftrightarrow(\mathsf{RLD})_{\mathrm{in}}\,f\sqcap(\mathsf{RLD})_{\mathrm{in}}\uparrow^{\mathsf{FCD}}Y\neq 0\Leftarrow(\mathsf{RLD})_{\mathrm{in}}\,f\sqcap(\mathsf{RLD})_{\mathrm{out}}\uparrow^{\mathsf{FCD}}Y\neq 0\Leftrightarrow(\mathsf{RLD})_{\mathrm{in}}\,f\sqcap\uparrow^{\mathsf{RLD}}Y\neq 0\Leftrightarrow(\mathsf{RLD})_{\mathrm{in}}\,(\mathsf{FCD})g\sqcap\uparrow^{\mathsf{RLD}}Y\neq 0\Leftarrow g\sqcap\uparrow^{\mathsf{RLD}}Y\neq 0\Leftrightarrow g\not\asymp\uparrow^{\mathsf{RLD}}Y$. □

Proposition 11. Let f be a funcoid. Then $V\circ M\circ V^{-1}\in\operatorname{GR}\langle f\times f\rangle^{*}M$ for every $V\in\operatorname{GR}f$.

Proof. $V\circ M\circ V^{-1}\in\operatorname{GR}(f\circ\uparrow M\circ f^{-1})=\operatorname{GR}\left\langle f\times^{(C)}f\right\rangle\uparrow M\supseteq\operatorname{GR}\langle f\times f\rangle\uparrow M=\operatorname{GR}\langle f\times f\rangle^{*}M$. [FIXME: Wrong direction of $\supseteq$.]

Because

$\uparrow^{\mathsf{FCD}}X\not\asymp\left\langle f\times^{(C)}f\right\rangle\uparrow^{\mathsf{FCD}}M\Leftrightarrow\uparrow^{\mathsf{RLD}}X\not\asymp\langle f\times f\rangle\uparrow^{\mathsf{RLD}}M\Leftrightarrow(\mathsf{FCD})(\uparrow^{\mathsf{RLD}}X\sqcap\langle f\times f\rangle\uparrow^{\mathsf{RLD}}M)\neq 0\Rightarrow(\mathsf{FCD})\uparrow^{\mathsf{RLD}}X\sqcap(\mathsf{FCD})\langle f\times f\rangle\uparrow^{\mathsf{RLD}}M\neq 0\Leftrightarrow(\mathsf{FCD})\uparrow^{\mathsf{RLD}}X\not\asymp(\mathsf{FCD})\langle f\times f\rangle\uparrow^{\mathsf{RLD}}M\Leftrightarrow\uparrow^{\mathsf{FCD}}X\not\asymp(\mathsf{FCD})\langle f\times f\rangle\uparrow^{\mathsf{RLD}}M$;

$\left\langle f\times^{(C)}f\right\rangle\uparrow^{\mathsf{FCD}}M\sqsubseteq(\mathsf{FCD})\langle f\times f\rangle\uparrow^{\mathsf{RLD}}M$

$\operatorname{GR}\left\langle f\times^{(C)}f\right\rangle\uparrow^{\mathsf{FCD}}M\supseteq\operatorname{GR}\,(\mathsf{FCD})\langle f\times f\rangle\uparrow^{\mathsf{RLD}}M\supseteq\operatorname{GR}\langle f\times f\rangle\uparrow^{\mathsf{RLD}}M$ □

Proposition 12. $\langle f\times f\rangle^{*}M\sqsubseteq g\circ\uparrow^{\mathsf{RLD}}M\circ g^{-1}$ whenever $(\mathsf{FCD})g=f$ for a reloid g.

Proof. For every $V\in\operatorname{GR}g$ we have $V\circ M\circ V^{-1}\in\operatorname{GR}\langle f\times f\rangle^{*}M$. Thus $g\circ\uparrow^{\mathsf{RLD}}M\circ g^{-1}=\bigsqcap\{V\circ M\circ V^{-1}\mid V\in\operatorname{GR}g\}\sqsupseteq\bigsqcap\operatorname{GR}\langle f\times f\rangle^{*}M=\operatorname{GR}\langle f\times f\rangle^{*}M$. □

Corollary 13. $\langle f\times f\rangle^{*}M\sqsubseteq\left\langle f\times^{(C)}f\right\rangle^{*}M$.

Corollary 14. $V\circ V^{-1}\in\operatorname{GR}\langle f\times f\rangle^{*}\Delta$; $f\circ f^{-1}\sqsupseteq\langle f\times f\rangle^{*}\Delta$.

Proof. ?? □

Lemma 15. $\operatorname{Cor}\langle f\times f\rangle^{*}g\sqsubseteq\Delta$ if $(\mathsf{FCD})g=f$ where $(\mathsf{FCD})g=f$ for a T_1-separable reloid g.

Proof. ?? □

Remark 16. I attempted to generalize the below theorem more than the standard general topology theorem about correspondence of compact and uniform spaces, but haven't really succeeded much, as it appears to be needed that the reloid in question is reflexive, symmetric, and transitive, that is just a uniform space as in the standard general topology.

Theorem 17. Let f be a T_1-separable compact reflexive symmetric funcoid and g be a reloid such that

1. $(\mathsf{FCD})g=f$;

2. $g\circ g^{-1}\sqsubseteq g$.