Theorem 34. A symmetric transitive reloid is totally bounded iff its Cauchy space is totally bounded.

Proof.

- **⇒.** Let $\mathcal{F}$ be a proper filter on $\operatorname{Ob}\nu$ and let $a \in \operatorname{atoms}\mathcal{F}$. It's enough to prove that a is Cauchy.

  Let $D \in \operatorname{GR}\nu$. Let also $E \in \operatorname{GR}\nu$ is symmetric and $E \circ E \subseteq D$. There existsa finite subset $F \subseteq \operatorname{Ob}\nu$ such that $\langle E\rangle F = \operatorname{Ob}\nu$. Then obviously exists $x \in F$ such that $a \sqsubseteq \uparrow^{\operatorname{Ob}\nu}\langle E\rangle\{x\}$, but $\langle E\rangle\{x\} \times \langle E\rangle\{x\} = E^{-1} \circ (\{x\} \times \{x\}) \circ E \subseteq D$, thus $a \times^{\mathsf{RLD}} a \sqsubseteq \uparrow^{\mathsf{RLD}(\operatorname{Ob}\nu;\operatorname{Ob}\nu)} D$.

  Because D was taken arbitrary, we have $a \times^{\mathsf{RLD}} a \sqsubseteq \nu$ that is a is Cauchy.

- **⇐.** Suppose that Cauchy space associated with a reloid ν is totally bounded but the reloid ν isn't totally bounded. So there exists a $D \in \operatorname{GR}\nu$ such that $(\operatorname{Ob}\nu) \setminus \langle D\rangle F \neq \emptyset$ for every finite set F.

  Consider the filter base
  $$S = \{(\operatorname{Ob}\nu) \setminus \langle D\rangle F \mid F \in \mathscr{P}\operatorname{Ob}\nu, F \text{ is finite}\}$$
  and the filter $\mathcal{F} = \prod \langle \uparrow^{\operatorname{Ob}\nu}\rangle S$ generated by this base. The filter $\mathcal{F}$ is proper because intersection $P \cap Q \in S$ for every $P, Q \in S$ and $\emptyset \notin S$. Thus there exists a Cauchy (for our Cauchy space) filter $\mathcal{X} \sqsubseteq \mathcal{F}$ that is $\mathcal{X} \times^{\mathsf{RLD}} \mathcal{X} \sqsubseteq \nu$.

  Thus there exists $M \in \mathcal{X}$ such that $M \times M \subseteq D$. Let F be a finite subset of $\operatorname{Ob}\nu$. Then $(\operatorname{Ob}\nu) \setminus \langle D\rangle F \in \mathcal{F} \sqsupseteq \mathcal{X}$. Thus $M \not\asymp (\operatorname{Ob}\nu) \setminus \langle D\rangle F$ and so there exists a point $x \in M \cap ((\operatorname{Ob}\nu) \setminus \langle D\rangle F)$.

  $\langle M \times M\rangle\{p\} \subseteq \langle D\rangle\{x\}$ for every $p \in M$; thus $M \subseteq \langle D\rangle\{x\}$.

  So $M \subseteq \langle D\rangle(F \cup \{x\})$. But this means that $M \in \mathcal{X}$ does not intersect $(\operatorname{Ob}\nu) \setminus \langle D\rangle(F \cup \{x\}) \in \mathcal{F} \sqsupseteq \mathcal{X}$, what is a contradiction (taken into account that $\mathcal{X}$ is proper). □

http://math.stackexchange.com/questions/104696/pre-compactness-total-boundedness-and-cauchy-sequential-compactness

10 Totally bounded funcoids

Definition 35. A funcoid ν is totally bounded iff

$$\forall X \in \operatorname{Ob}\nu \exists \mathcal{X} \in \mathfrak{F}^{\operatorname{Ob}\nu}: (0 \neq \mathcal{X} \sqsubseteq \uparrow^{\operatorname{Ob}\nu} X \wedge \mathcal{X} \times^{\mathsf{FCD}} \mathcal{X} \sqsubseteq \nu).$$

This can be rewritten in elementary terms (without using funcoidal product:

$\mathcal{X} \times^{\mathsf{FCD}} \mathcal{X} \sqsubseteq \nu \Leftrightarrow \forall P \in \partial\mathcal{X}: \mathcal{X} \sqsubseteq \langle\nu\rangle P \Leftrightarrow \forall P \in \partial\mathcal{X}, Q \in \partial\mathcal{X}: P\ [\nu]^*\ Q \Leftrightarrow \forall P, Q \in \operatorname{Ob}\nu: (\forall E \in \mathcal{X}: (E \cap P \neq \emptyset \wedge E \cap Q \neq \emptyset) \Rightarrow P\ [\nu]^*\ Q)$.

Note that probably I am the first person which has written the above formula (for proximity spaces for instance) explicitly.

11 On principal low filter spaces

Definition 36. A low filter space $(U; \mathscr{C})$ is *principal* when all filters in $\mathscr{C}$ are principal.

Definition 37. A low filter space $(U; \mathscr{C})$ is *reflexive* when $\forall x \in U: \uparrow^U\{x\} \in \mathscr{C}$.

Proposition 38. Having fixed a set U, principal reflexive low filter spaces on U bijectively correspond to principal reflexive symmertic endoreloids on U.

Proof. ??

http://math.stackexchange.com/questions/701684/union-of-cartesian-squares □