Cauchy Filters on Reloids

BY VICTOR PORTON

Email: porton@narod.ru
Web: http://www.mathematics21.org

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Abstract

In this article I consider *low filters* on reloids, generalizing Cauchy filters on uniform spaces. Using low filters, I define Cauchy-complete reloids, generalizing complete uniform spaces.

1 Preface

This is a preliminary partial draft.

To understand this article you need first look into my book [1].

As my book is yet in preprint stage and I may change it, I probably will integrate the content of this article into the book.

http://math.stackexchange.com/questions/401989/what-are-interesting-properties-of-totally-bounded-uniform-spaces

http://ncatlab.org/nlab/show/proximity+space#uniform_spaces for a proof sketch that proximities correspond to totally bounded uniformities.

2 Low filters space

Definition 1. A *lower set*[1] *of proper filters* on U (a set) is a set $\mathscr{C}$ of proper filters on U, such that if $0 \neq \mathcal{G} \sqsubseteq \mathcal{F}$ and $\mathcal{F} \in \mathscr{C}$ then $\mathcal{G} \in \mathscr{C}$. [TODO: Probably should include the improper filter.]

Definition 2. I call *low filters space* a set together with a lower set of proper filters on this set.

Definition 3. $\mathrm{PR}(U; \mathscr{C}) = \mathscr{C}$; $\mathrm{Ob}(U; \mathscr{C}) = U$.[2]

Definition 4. Introduce an order on low filters spaces: $(U; \mathscr{C}) \sqsubseteq (U; \mathscr{D}) \Leftrightarrow \mathscr{C} \sqsubseteq \mathscr{D}$.

3 Cauchy spaces

Definition 5. A *Cauchy space* on a set X is a low filters space $(U; \mathscr{C})$ (element of $\mathscr{C}$ are called *Cauchy filters*) such that:

1. $\forall x \in U: \uparrow^X \{x\} \in \mathscr{C}$;
2. If $\mathcal{F}$, $\mathcal{G}$ are Cauchy filters and $\mathcal{F} \not\asymp \mathcal{G}$ then $\mathcal{F} \sqcup \mathcal{G}$ is a Cauchy filter.

Definition 6. A *completely Cauchy space* on a set X is a low filters space $(U; \mathscr{C})$ (element of $\mathscr{C}$ are called *Cauchy filters*) such that:

1. $\forall x \in X: \uparrow^X \{x\} \in \mathscr{C}$;
2. If S is a nonempty set of Cauchy filters and $\bigsqcap S \neq 0^{\mathfrak{F}(X)}$ then $\bigsqcup S$ is a Cauchy filter.

1. *Remember that our orders on filters is the reverse to set theoretic inclusion. It could be called an* upper *set in other sources.*

2. PR is from English word *profile*.