Certain categories are cartesian closed

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November 25, 2013

Abstract

I prove that the category of continuous maps between endofuncoids is cartesian closed. Whether the category of continuous maps between endoreloids is cartesian closed is yet an open problem.

This is a rough draft. There are errors!

Cartesian closed categories

Definition 1. A category is *cartesian closed* iff:

- It has finite products.
- For each objects A, B is given an object $\mathrm{MOR}(A;B)$ (*exponentiation*) and a morphism $\varepsilon^{\mathbf{Dig}}_{A,B}\colon \mathrm{MOR}(A;B)\times A\to B$.
- For each morphism $f\colon Z\times A\to B$ there is given a morphism (*exponential transpose*) $\sim f\colon Z\to \mathrm{MOR}(A;B)$.
- $\varepsilon\circ(\sim f\times 1_{\mathrm{A}})=f$.
- $\sim(\varepsilon\circ(g\times 1_A))=g$.

Our puspose is to prove (or disprove) that categories **Dig**, **Fcd**, and **Rld** are cartesian closed. Note that they have finite (and even infinite) products is already proved in http://www.mathematics21.org/binaries/product.pdf

Definitions of our categories

Categories **Dig**, **Fcd**, and **Rld** are respectively categories of:

1. discretely continuous maps between digraphs;
2. (proximally) continuous maps between endofuncoids;
3. (uniformly) continuous maps between endoreloids.

Definition 2. *Digraph* is an endomorphism of the category **Rel**.

Definition 3. Category **Dig** of digraphs is the category whose objects are digraphs and morphisms are discretely continuous maps between digraphs. That is morphisms from a digraph μ to a digraph ν are functions (or more precisely morphisms of **Set**) f such that $f\circ\mu\sqsubseteq\nu\circ f$ (or equivalently $\mu\sqsubseteq f^{-1}\circ\nu\circ f$ or equivalently $f\circ\mu\circ f^{-1}\sqsubseteq\nu$).