- $\exists n \in \mathbb{N} \colon \mathrm{totBound}_\beta(f^n)$
- $\exists n \in \mathbb{N} \colon \mathrm{totBound}_\alpha(f^0 \sqcup \ldots \sqcup f^n)$
- $\exists n \in \mathbb{N} \colon \mathrm{totBound}_\beta(f^0 \sqcup \ldots \sqcup f^n)$
- $\mathrm{totBound}_\alpha(f^0 \sqcup f^1 \sqcup f^2 \sqcup \ldots)$
- $\mathrm{totBound}_\beta(f^0 \sqcup f^1 \sqcup f^2 \sqcup \ldots)$

Some of the above defined predicates are equivalent:

Proposition 23.

- $\forall E \in \mathrm{GR}\, f\, \exists n \in \mathbb{N} \colon \mathrm{thick}_\alpha(E^n) \Leftrightarrow \exists n \in \mathbb{N} \colon \mathrm{totBound}_\alpha(f^n)$.
- $\forall E \in \mathrm{GR}\, f\, \exists n \in \mathbb{N} \colon \mathrm{thick}_\beta(E^n) \Leftrightarrow \exists n \in \mathbb{N} \colon \mathrm{totBound}_\beta(f^n)$.

Proof. Because every $F \in \mathrm{GR}\, f^n$ is a superset of E^n for some $E \in \mathrm{GR}\, f$. □

Proposition 24.

- $\forall E \in \mathrm{GR}\, f\, \exists n \in \mathbb{N} \colon \mathrm{thick}_\alpha(E^0 \cup \ldots \cup E^n) \Leftrightarrow \exists n \in \mathbb{N} \colon \mathrm{totBound}_\alpha(f^0 \sqcup \ldots \sqcup f^n)$.
- $\forall E \in \mathrm{GR}\, f\, \exists n \in \mathbb{N} \colon \mathrm{thick}_\beta(E^0 \cup \ldots \cup E^n) \Leftrightarrow \exists n \in \mathbb{N} \colon \mathrm{totBound}_\beta(f^0 \sqcup \ldots \sqcup f^n)$.

Proof. $f^0 \sqcup \ldots \sqcup f^n = f^0 \cap \ldots \cap f^n$. Thus every $F \in \mathrm{GR}(f^0 \cap \ldots \cap f^n)$ we have $F \in f^k$, thus $F \supseteq E_k^k$ for all k for some $E_k \in \mathrm{GR}\, f$ and so $F \supseteq E^0 \cup \ldots \cup E^n$ where $E = E_0 \cap \ldots \cap E_k \in \mathrm{GR}\, f$. □

Proposition 25. All predicates in the above list are pairwise equivalent in the case if f is a uniform space.

Proof. Because $f \circ f = f$. □