Proof. Let φ is the monovalued, surjective function, which induces the reloid f.

We have $\mu \sqsubseteq f^{-1} \circ \nu \circ f$.

Let $F \in \operatorname{GR} \nu$. Then there exists $E \in \operatorname{GR} \mu$ such that $E \subseteq \varphi^{-1} \circ F \circ \varphi$.

Since μ is β-totally bounded, there exists a finite subset A of $\operatorname{Ob} \mu$ such that $\langle E \rangle A = \operatorname{Ob} \mu$.

We claim $\langle F \rangle \langle \varphi \rangle A = \operatorname{Ob} \nu$.

Indeed let $y \in \operatorname{Ob} \nu$ be an arbitrary point. Since φ is surjective, there exists $x \in \operatorname{Ob} \mu$ such that $\varphi x = y$. Since $\langle E \rangle A = \operatorname{Ob} \mu$ there exists $a \in A$ such that $a \mathrel{E} x$ and thus $a \mathrel{(\varphi^{-1} \circ F \circ \varphi)} x$. So $(\varphi a; y) = (\varphi a; \varphi x) \in F$. Therefore $y \in \langle F \rangle \langle \varphi \rangle A$. □

Theorem 20. Let μ and ν are endoreloids. Let f is a principal $\mathrm{C}''(\mu; \nu)$ continuous, surjective reloid. Then if μ is α-totally bounded then ν is also α-totally bounded.

Proof. Let φ is the surjective binary relation which induces the reloid f.

We have $f \circ \mu \circ f^{-1} \sqsubseteq \nu$.

Let $F \in \operatorname{GR} \nu$. Then there exists $E \in \operatorname{GR} \mu$ such that $\varphi \circ E \circ \varphi^{-1} \subseteq F$.

There exists a finite cover S of $\operatorname{Ob} \mu$ such that

$$\bigcup \{A \times A \mid A \in S\} \subseteq E.$$

Thus $\varphi \circ (\bigcup \{A \times A \mid A \in S\}) \circ \varphi^{-1} \subseteq F$ that is $\bigcup \{\langle \varphi \rangle A \times \langle \varphi \rangle A \mid A \in S\} \subseteq F$.

It remains to prove that $\{\langle \varphi \rangle A \mid A \in S\}$ is a cover of $\operatorname{Ob} \nu$. It is true because φ is a surjection and S is a cover of $\operatorname{Ob} \mu$. □

A stronger statement (principality requirement removed):

Conjecture 21. The image of a uniformly continuous entirely defined monovalued surjective reloid from a (α-, β-)totally bounded endoreloid is also (α-, β-)totally bounded.

Can we remove the requirement to be entirely defined from the above conjecture?

Question 22. Under which conditions it's true that join of (α-, β-) totally bounded reloids is also totally bounded?

Additional predicates

We may consider also the following predicates expressing different kinds of what is intuitively is understood as boundness. Their usefulness is unclear, but I present them for completeness.

- $\forall E \in \operatorname{GR} f \exists n \in \mathbb{N} \colon \operatorname{thick}_\alpha(E^n)$
- $\forall E \in \operatorname{GR} f \exists n \in \mathbb{N} \colon \operatorname{thick}_\beta(E^n)$
- $\forall E \in \operatorname{GR} f \exists n \in \mathbb{N} \colon \operatorname{thick}_\alpha(E^0 \cup \ldots \cup E^n)$
- $\forall E \in \operatorname{GR} f \exists n \in \mathbb{N} \colon \operatorname{thick}_\beta(E^0 \cup \ldots \cup E^n)$
- $\exists n \in \mathbb{N} \colon \operatorname{totBound}_\alpha(f^n)$