**Proof.** Because $\operatorname{thick}_\alpha(E) \Rightarrow \operatorname{thick}_\beta(E)$. □

**Proposition 13.** If an endoreloid $f$ is reflexive and $\operatorname{Ob} f$ is finite then $f$ is both $\alpha$-totally bounded and $\beta$-totally bounded.

**Proof.** It enough to prove that $f$ is $\alpha$-totally bounded. Really, every $E \in \operatorname{xyGR} f$ is reflexive. Thus $\{x\} \times \{x\} \subseteq E$ for $x \in \operatorname{Ob} f$ and thus $\{\{x\} \mid x \in \operatorname{Ob} f\}$ is a sought for finite cover of $\operatorname{Ob} f$. □

**Obvious 14.**

- A principal endo-reloid induced by a **Rel**-morphism $E$ is $\alpha$-totally bounded iff $E$ is $\alpha$-thick.
- A principal endo-reloid induced by a **Rel**-morphism $E$ is $\beta$-totally bounded iff $E$ is $\beta$-thick.

**Example 15.** There is a $\beta$-totally bounded endoreloid which is not $\alpha$-totally bounded.

**Proof.** It follows from the example above and properties of principal endoreloids. □

## Special case of uniform spaces

**Definition 16.** *Uniform space* is essentially the same as symmetric, reflexive and transitive endoreloid.

**Exercise 1.** Prove that it is essentially the same as the standard definition of a uniform space (see Wikipedia or PlanetMath).

**Theorem 17.** Let $f$ is such a endoreloid that $f \circ f^{-1} \sqsubseteq f$. Then $f$ is $\alpha$-totally bounded iff it is $\beta$–totally bounded.

**Proof.**

**⇒.** Proved above.

**⇐.** For every $\varepsilon \in \operatorname{GR} f$ we have that $\langle\varepsilon\rangle\{c_0\}, \dots, \langle\varepsilon\rangle\{c_n\}$ covers the space. $\langle\varepsilon\rangle\{c_i\} \times \langle\varepsilon\rangle\{c_i\} \subseteq \varepsilon \circ \varepsilon^{-1}$ because for $x \in \langle\varepsilon\rangle\{c_i\}$ (the same as $c_i \in \langle\varepsilon^{-1}\rangle\{x\}$) we have $\langle\langle\varepsilon\rangle\{c_i\} \times \langle\varepsilon\rangle\{c_i\}\rangle\{x\} = \langle\varepsilon\rangle\{c_i\} \subseteq \langle\varepsilon\rangle\langle\varepsilon^{-1}\rangle\{x\} = \langle\varepsilon \circ \varepsilon^{-1}\rangle\{x\}$. There exists $\varepsilon' \in \operatorname{GR} f$ such that $\varepsilon \circ \varepsilon^{-1} \subseteq \varepsilon'$ because $f \circ f^{-1} \sqsubseteq f$. Thus for every $\varepsilon'$ we have $\langle\varepsilon\rangle\{c_i\} \times \langle\varepsilon\rangle\{c_i\} \subseteq \varepsilon'$ and so

$$\langle\varepsilon\rangle\{c_0\}, \dots, \langle\varepsilon\rangle\{c_n\}.$$

is a sought for finite cover. □

**Corollary 18.** A uniform space is $\alpha$-totally bounded iff it is $\beta$-totally bounded.

**Proof.** From the theorem and the definition of uniform spaces. □

## Relationships with other properties

**Theorem 19.** Let $\mu$ and $\nu$ are endoreloids. Let $f$ is a principal $\mathrm{C}'(\mu;\nu)$ continuous, monovalued, surjective reloid. Then if $\mu$ is $\beta$-totally bounded then $\nu$ is also $\beta$-totally bounded.