Proof. Let $\mathrm{thick}_\alpha(E)$. Then there exists a finite cover S of the set $\mathrm{Ob}\, E$ such that $\forall A \in S$: $A \times A \subseteq \mathrm{GR}\, E$. Without loss of generality assume $A \neq \emptyset$ for every $A \in S$. So $A \subseteq \langle E \rangle \{x_A\}$ for some x_A for every $A \in S$. So $\langle E \rangle \{x_A \mid A \in S\} = \bigcup \{\langle E \rangle \{x_A\} \mid A \in S\} = \mathrm{Ob}\, E$ and thus E is β-thick. $\square$

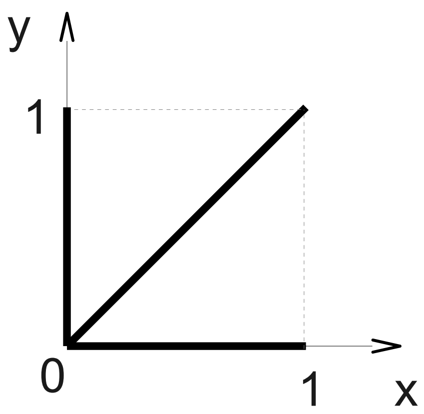
Obvious 7. Let X be a set, A and B are **Rel**-endorelations on X and $B \sqsupseteq A$. Then:

- $\mathrm{thick}_\alpha(A) \Rightarrow \mathrm{thick}_\alpha(B)$;
- $\mathrm{thick}_\beta(A) \Rightarrow \mathrm{thick}_\beta(B)$.

Example 8. There is a β-thick **Rel**-morphism which is not α-thick.

Proof. Consider the **Rel**-morphism on $[0; 1]$ with the below graph:

$$\Gamma = \{(x; x) \mid x \in [0; 1]\} \cup \{(x; 0) \mid x \in [0; 1]\} \cup \{(0; x) \mid x \in [0; 1]\}.$$



Γ is β-chick because $\langle \Gamma \rangle \{0\} = [0; 1]$.

To prove that Γ is not α-thick it's enough to prove that every set A such that $A \times A \subseteq \Gamma$ is finite.

Suppose for the contrary that A is infinite. Then A contains more than one non-zero points y, z ($y \neq z$). Without loss of generality $y < z$. So we have that (y, z) is not of the form $(y; y)$ nor $(0; y)$ nor $(y; 0)$. Therefore $A \times A$ isn't a subset of Γ. $\square$

Totally bounded endo-reloids

The below is a straightforward generalization of the customary definition of totally bounded sets on uniform spaces (it's proved below that for uniform spaces the below definitions are equivalent).

Definition 9. An endoreloid f is *α-totally bounded* ($\mathrm{totBound}_\alpha(f)$) if every $E \in \mathrm{xyGR}\, f$ is α-thick.

Definition 10. An endoreloid f is *β-totally bounded* ($\mathrm{totBound}_\beta(f)$) if every $E \in \mathrm{xyGR}\, f$ is β-thick.

Remark 11. We could rewrite the above definitions in a more algebraic way like $\mathrm{xyGR}\, f \subseteq \mathrm{thick}_\alpha$ (with thick_α would be defined as a set rather than as a predicate), but we don't really need this simplification.

Proposition 12. If an endoreloid is α-totally bounded then it is β-totally bounded.