Total boundness of reloids

by Victor Porton

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Abstract

I generalize total boundness of uniform spaces for arbitrary reloids (filters on a cartesian product of sets). For reloids total boundness splits into two different concepts: α-total boundness and β-total boundness.

Notation

I call *reloid* a triple $(A; B; \mathcal{F})$ where A, B are sets and $\mathcal{F}$ is a filter on the cartesian product $A \times B$ of these sets. I will denote $\mathrm{GR}(A; B; \mathcal{F}) = \mathcal{F}$ for a reloid $(A; B; \mathcal{F})$. Reloids are a generalization of uniform spaces. *Source* of a reloid is $\mathrm{Src}(A; B; \mathcal{F}) = A$ and destination $\mathrm{Dst}(A; B; \mathcal{F}) = B$. I also denote $\mathrm{xyGR}(A; B; F) = \{(A; B; f) \mid f \in F\}$.

I will refer to a pair $f = (U; E)$ of a (small) set U and a binary relation $E \subseteq E \times E$ as **Rel**-endomorphism. Furthermore $\mathrm{Ob}\, f = U$ and $\mathrm{GR}\, f = E$.

I denote $\langle E \rangle X = \{y \mid \exists x \in X : x\, E\, y\}$ for a binary relation E and set X and $\langle E \rangle X = \langle \mathrm{GR}\, E \rangle X$ for a **Rel**-endomorphism E.

I define composition of reloids $(B; C; G) \circ (A; B; F) = (A; C; H)$ where H is the filter induced by the filter base $\{g \circ f \mid f \in F, g \in G\}$.

The reverse reloid is defined by the formula $(A; B; F)^{-1} = (B; A; F^{-1})$.

I define partial order on the set of reloids as $(A; B; F) \sqsubseteq (A; B; G) \Leftrightarrow F \supseteq G$. The set of reloids with given source and destination is a complete lattice with join denoted $\sqcup$ and meet denotes $\sqcap$.

See http://www.mathematics21.org/algebraic-general-topology.html for more details.

Thick binary relations

Definition 1. I will call *α-thick* and denote $\mathrm{thick}_\alpha(E)$ a **Rel**-endomorphism E when there exists a finite cover S of $\mathrm{Ob}\, E$ such that $\forall A \in S : A \times A \subseteq \mathrm{GR}\, E$.

Definition 2. $\mathrm{CS}(S) = \bigcup \{A \times A \mid A \in S\}$ for a collection S of sets.

Remark 3. CS means "cartesian squares".

Obvious 4. A **Rel**-endomorphism is *α-thick* iff there exists a finite cover S of $\mathrm{Ob}\, E$ such that $\mathrm{CS}(S) \subseteq \mathrm{GR}\, E$.

Definition 5. I will call *β-thick* and denote $\mathrm{thick}_\beta(E)$ a **Rel**-endomorphism E when iff there exists a finite set B such that $\langle E \rangle B = \mathrm{Ob}\, E$.

Proposition 6. $\mathrm{thick}_\alpha(E) \Rightarrow \mathrm{thick}_\beta(E)$.