Backward Funcoids

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This is a preliminary partial draft.

Fix a family $\mathfrak{A}$ of posets.

Definition 1. Let f be a staroid of filters $\mathfrak{F}(\mathfrak{A}_i)$ on boolean lattices $\mathfrak{A}_i$. *Backward funcoid* for the argument $k \in \operatorname{dom} \mathfrak{A}$ of f is the funcoid $\operatorname{Back}(f; k)$ defined by the formula (for every $X \in \mathfrak{A}_k$)

$$\langle \operatorname{Back}(f; k) \rangle X = \left\{ L \in \prod_{i \in \operatorname{dom} \mathfrak{A}} \mathfrak{F}(\mathfrak{A}_i) \mid X \in \langle f \rangle_k L \right\}.$$

Proposition 2. Backward funcoid is properly defined.

Proof. $\langle \operatorname{Back}(f; k) \rangle^\ast (X \sqcup Y) = \{ L \in \prod \mathfrak{A} \mid X \sqcup Y \in \langle f \rangle_k L \} = \{ L \in \prod \mathfrak{A} \mid X \in \langle f \rangle_k L \vee Y \in \langle f \rangle_k L \} = \{ L \in \prod \mathfrak{A} \mid X \in \langle f \rangle_k L \} \cup \{ L \in \prod \mathfrak{A} \mid Y \in \langle f \rangle_k L \} = \langle \operatorname{Back}(f; k) \rangle^\ast X \cup \langle \operatorname{Back}(f; k) \rangle^\ast Y$. □

Obvious 3. Backward funcoid is co-complete.

Proposition 4. If f is a principal staroid then $\operatorname{Back}(f; k)$ is a complete funcoid. [TODO: generalize for boolean lattices?]

Proof. ?? □

Proposition 5. f can be restored from $\operatorname{Back}(f; k)$ (for every fixed k).

Proof. ?? □

Proposition 6. $f \mapsto \operatorname{Back}(f; k)$ is an order isomorphism $\mathsf{Strd}^{\mathfrak{A}} \to \mathsf{FCD}\big(\mathfrak{A}_k; \mathsf{Strd}^{i \in (\operatorname{dom} \mathfrak{A}) \setminus \{k\}}\big)$.

Proof. ?? □