for every small n-ary relation f where n is an ordinal number and $i \in n$. Particularly for every n-ary relation f and $i \in n$ where $n \in \mathbb{N}$.

$$\Pr_i f = \{x_i \mid [\![x_0, \ldots, x_{n-1}]\!] \in f\}.$$

Recall that cartesian product is defined as follows:

$$\prod a = \left\{z \in \left(\bigcup \operatorname{im} a\right)^{\operatorname{dom} a} \mid \forall i \in \operatorname{dom} a: z(i) \in a_i\right\}.$$

Obvious 3. If a is a small function, then $\prod a = \{z \in \mho^{\operatorname{dom} a} \mid \forall i \in \operatorname{dom} a: z(i) \in a_i\}$.

2.1 Currying and uncurrying

2.1.1 The customary definition

Let X, Y, Z are sets.

We will consider variables $x \in X$ and $y \in Y$.

Let a function $f \in Z^{X \times Y}$. Then $\operatorname{curry}(f) \in (Z^Y)^X$ is the function defined by the formula $(\operatorname{curry}(f)x)\, y = f(x; y)$.

Let now $f \in (Z^Y)^X$. Then $\operatorname{uncurry}(f) \in Z^{X \times Y}$ is the function defined by the formula $\operatorname{uncurry}(f)(x; y) = (f x)\, y$.

Obvious 4.

1. $\operatorname{uncurry}(\operatorname{curry}(f)) = f$ for every $f \in Z^{X \times Y}$.
2. $\operatorname{curry}(\operatorname{uncurry}(f)) = f$ for every $f \in (Z^Y)^X$.

2.1.2 Currying and uncurrying with a dependent variable

Let X, Z are sets and Y is a function with the domain X. (Vaguely saying, Y is a variable dependent on X.)

The disjoint union $\coprod Y = \bigcup \{\{i\} \times Y_i \mid i \in \operatorname{dom} Y\} = \{(i; x) \mid i \in \operatorname{dom} Y, x \in Y_i\}$.

We will consider variables $x \in X$ and $y \in Y_x$.

Let a function $f \in Z^{\coprod_{i \in X} Y_i}$ (or equivalently $f \in Z^{\coprod Y}$). Then $\operatorname{curry}(f) \in \prod_{i \in X} Z^{Y_i}$ is the function defined by the formula $(\operatorname{curry}(f)x)\, y = f(x; y)$.

Let now $f \in \prod_{i \in X} Z^{Y_i}$. Then $\operatorname{uncurry}(f) \in Z^{\coprod_{i \in X} Y_i}$ is the function defined by the formula $\operatorname{uncurry}(f)(x; y) = (f x)\, y$.

Obvious 5.

1. $\operatorname{uncurry}(\operatorname{curry}(f)) = f$ for every $f \in Z^{\coprod_{i \in X} Y_i}$.
2. $\operatorname{curry}(\operatorname{uncurry}(f)) = f$ for every $f \in \prod_{i \in X} Z^{Y_i}$.

Remark 6. There is nothing said anything about currying with dependent variables in Wikipedia. Am I really the first person who formulated this simple generalization of currying and uncurrying?

2.2 Functions with ordinal numbers of arguments

Let Ord is the set of small ordinal numbers.

If X and Y are sets and n is an ordinal number, the set of functions taking n arguments on the set X and returning a value in Y is Y^{X^n}.

The set of all small functions taking ordinal numbers of arguments is $Y^{\bigcup_{n \in \mathrm{Ord}} X^n}$.

I will denote $\operatorname{OrdVar}(X) = \mho^{\bigcup_{n \in \mathrm{Ord}} X^n}$ and call it *ordinal variadic*. ("Var" in this notation is taken from the word *variadic* in the collocation *variadic function* used in computer science.)

3 On sums of ordinals

Let a is an ordinal-indexed family of ordinals.