# A New Kind of Product of Ordinal Number of Relations having Ordinal Numbers of Arguments

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**Abstract**

Infinite associativity is defined for functions taking an ordinal numbers of arguments. As an important example of an infinite associative function I define *ordinated product* and research it's properties. Ordinated product is an infinitely associative function.

**Keywords:** infinite associativity, abstract algebra, set theory, ordinal numbers, product, relations

**A.M.S. subject classification:** 05C25, 03E10

## 1 Introduction

We will consider some function $f$ which takes an arbitrary ordinal number of arguments. That is $f$ can be taken for arbitary (small, if to be precise) ordinal number of arguments. More formally: Let $x = x_{i \in n}$ is a family indexed by an ordinal $n$. Then $f(x)$ can be taken. The same function $f$ can take different number of arguments. (See below for the exact definition.)

Some of such functions $f$ are associative in the sense defined below. If a function is associative in the below defined sense, then the binary operation induced by this function is associative in the usual meaning of the word "associativity" as defined in basic algebra.

I also introduce and research an important example of infinitely associative function, which I call *ordinated product*.

Note that my searching about infinite associativity and ordinals in Internet has provided no useful results. As such there is a reason to assume that my research of generalized associativity in terms of ordinals is novel.

## 2 Used notation

We identify natural numbers with finite Von Neumann's ordinals (further just *ordinals* or *ordinal numbers*).

For simplicity we will deal with small sets (members of a Grothendieck universe). We will denote the Grothendieck universe (aka *universal set*) as $\mho$.

$(\lambda x \in D: f(x)) \overset{\text{def}}{=} \{(x; f(x)) \mid x \in D\}$ for every set $D$ and a form $f$ taking $x$ as argument.

I will denote a tuple of $n$ elements like $[\![a_0; \ldots; a_{n-1}]\!]$. By definition

$$[\![a_0; \ldots; a_{n-1}]\!] = \{(0; a_0), \ldots, (n-1; a_{n-1})\}.$$

Note that an ordered pair $(a; b)$ is not the same as the tuple $[\![a; b]\!]$ of two elements.

**Definition 1.** An *anchored relation* is a tuple $[\![n; r]\!]$ where $n$ is an index set and $r$ is an $n$-ary relation.

For an anchored relation arity $[\![n; r]\!] = n$. The *graph*[1] of $[\![n; r]\!]$ is defined as follows: $\mathrm{GR}[\![n; r]\!] = r$.

**Definition 2.** $\mathrm{Pr}_i$ is a small function defined by the formula

$$\mathrm{Pr}_i\, f = \{x_i \mid x \in f\}$$

1. It is unrelated with graph theory.