$x\times^{\mathsf{FCD}} y\in\operatorname{atoms}\bigsqcap^{\mathsf{FCD}}(\Gamma(A;B)\cap\operatorname{GR} f)\Leftrightarrow\forall K\in\Gamma(A;B)\cap\operatorname{GR} f: x\times^{\mathsf{FCD}} y\in\operatorname{atoms} K\Leftarrow\forall K\in\operatorname{GR} f: x\times^{\mathsf{FCD}} y\in\operatorname{atoms} K\Leftrightarrow x\times^{\mathsf{RLD}} y\not\asymp f$.

??

It's enough to prove $\operatorname{up}^{\Gamma(A;B)}(\mathsf{FCD})\, f=\Gamma(A;B)\cap\operatorname{GR} f$.

Really, $\operatorname{up}^{\Gamma(A;B)}(\mathsf{FCD})\, f=\operatorname{up}^{\Gamma(A;B)}\bigsqcap^{\mathsf{FCD}}\operatorname{GR} f$

$K\in\operatorname{up}^{\Gamma(A;B)}\bigsqcap^{\mathsf{FCD}}\operatorname{GR} f\Rightarrow K\sqsupseteq\bigsqcap^{\mathsf{FCD}}\operatorname{GR} f$ [TODO: To continue this implication we MUST use that $K\in\Gamma$ (otherwise take thick identity as a counterexample)]

?? □

Theorem 55. *$b\not\asymp^{\operatorname{Anch}(\mathfrak{A})}\operatorname{StarComp}(a;f)\Leftrightarrow\forall A\in\operatorname{GR} a, B\in\operatorname{GR} b, i\in n: A_i\,[f_i]\,B_i$ for anchored relations a and b, provided that $\operatorname{Src} f_i$ are atomic posets.*

Proof.

$$\begin{aligned}
& b\not\asymp^{\operatorname{Anch}(\mathfrak{A})}\operatorname{StarComp}(a;f) && \Leftrightarrow\\
& \exists x\in\operatorname{Anch}(\mathfrak{A})\setminus\{\bot\}:(x\sqsubseteq b\wedge x\sqsubseteq\operatorname{StarComp}(a;f)) && \Leftrightarrow\\
& \exists x\in\operatorname{Anch}(\mathfrak{A})\setminus\{\bot\}:(x\sqsubseteq b\wedge\forall B\in\operatorname{GR} x: B\in\operatorname{GR}\operatorname{StarComp}(a;f)) && \Leftrightarrow\\
& \exists x\in\operatorname{Anch}(\mathfrak{A})\setminus\{\bot\}:(x\sqsubseteq b\wedge\forall B\in\operatorname{GR} x:(\lambda i\in n:\langle f_i{}^{-1}\rangle B_i)\in\operatorname{GR} a) && \Leftrightarrow && \text{remove?}\\
& \exists x\in\operatorname{Anch}(\mathfrak{A})\setminus\{\bot\}:(\forall B\in\operatorname{GR} x: B\in\operatorname{GR} b\wedge\forall B\in\operatorname{GR} x:(\lambda i\in n:\langle f_i{}^{-1}\rangle B_i)\in\operatorname{GR} a) && \Leftrightarrow\\
& \exists x\in\operatorname{Anch}(\mathfrak{A})\setminus\{\bot\}\,\forall B\in\operatorname{GR} x:(B\in\operatorname{GR} b\wedge(\lambda i\in n:\langle f_i{}^{-1}\rangle B_i)\in\operatorname{GR} a) && \Leftrightarrow\\
& ??\\
& \forall A\in\operatorname{GR} a, B\in\operatorname{GR} b, i\in n: A_i\not\asymp\langle f_i{}^{-1}\rangle B_i && \Leftrightarrow\\
& \forall A\in\operatorname{GR} a, B\in\operatorname{GR} b, i\in n: A_i\,[f_i]\,B_i. && \Leftrightarrow\\
& ??\\
& \exists x\in\operatorname{Anch}(\mathfrak{A})\setminus\{\bot\}:(\forall A\in\operatorname{GR} x:(A\in\operatorname{GR} a\wedge(\lambda i\in n:\langle f_i\rangle A_i)\in\operatorname{GR} b))\\
& \exists x\in\operatorname{Anch}(\mathfrak{A})\setminus\{\bot\}:(x\sqsubseteq a\wedge\forall A\in\operatorname{GR} x:(\lambda i\in n:\langle f_i\rangle A_i)\in\operatorname{GR} b)\\
& \exists x\in\operatorname{Anch}(\mathfrak{A})\setminus\{\bot\}:(x\sqsubseteq a\wedge\forall A\in\operatorname{GR} x: A\in\operatorname{GR}\operatorname{StarComp}(b;f^{\dagger}))\\
& \exists x\in\operatorname{Anch}(\mathfrak{A})\setminus\{\bot\}:(x\sqsubseteq a\wedge x\sqsubseteq\operatorname{StarComp}(b;f^{\dagger})) && \Leftrightarrow\\
& a\not\asymp^{\operatorname{Anch}(\mathfrak{A})}\operatorname{StarComp}(b;f^{\dagger})
\end{aligned}$$

□