Proof. Let $\mathcal{P} \in \operatorname{dom} S$, $\mathcal{Q} \in \operatorname{im} S$. Then there exist $\mathcal{P}'$ and $\mathcal{Q}'$ such that $(\mathcal{P}, \mathcal{Q}') \in S$, $(\mathcal{P}', \mathcal{Q}) \in S$. $\mathcal{P} \times \mathcal{Q}' \cap \mathcal{P}' \times \mathcal{Q} = (\mathcal{P} \cap \mathcal{P}') \times (\mathcal{Q} \cap \mathcal{Q}')$ (used the previous theorem). $\bigcap^{\mathfrak{F}}\{\mathcal{A} \times \mathcal{B} \mid (\mathcal{A}; \mathcal{B}) \in S\} \subseteq \mathcal{P} \times \mathcal{Q}' \cap \mathcal{P}' \times \mathcal{Q} = (\mathcal{P} \cap \mathcal{P}') \times (\mathcal{Q} \cap \mathcal{Q}') \subseteq \mathcal{P} \times \mathcal{Q}$. This implies that

$$\begin{aligned} \bigcap^{\mathfrak{F}}\{\mathcal{A} \times \mathcal{B} \mid (\mathcal{A}; \mathcal{B}) \in S\} &= \bigcap^{\mathfrak{F}}\{\mathcal{A} \times \mathcal{B} \mid \mathcal{A} \in \operatorname{dom} S, \mathcal{B} \in \operatorname{im} S\} \\ &= \bigcap^{\mathfrak{F}}\{A \times B \mid A \in \langle \mathrm{up} \rangle \operatorname{dom} S, B \in \langle \mathrm{up} \rangle \operatorname{im} S\}. \end{aligned}$$

On the other side

$\bigcap^{\mathfrak{F}} \operatorname{dom} S \times \bigcap^{\mathfrak{F}} \operatorname{im} S = \bigcap^{\mathfrak{F}}\{A \times B \mid A \in \mathrm{up} \bigcap^{\mathfrak{F}} \operatorname{dom} S, B \in \mathrm{up} \bigcap^{\mathfrak{F}} \operatorname{im} S\}$

??

If $A \in \langle \mathrm{up} \rangle \operatorname{dom} S$ then $A \in \mathrm{up} \bigcap^{\mathfrak{F}} \operatorname{dom} S$. If $A \in \mathrm{up} \bigcap^{\mathfrak{F}} \operatorname{dom} S$ then □

$\forall S \in \mathscr{P}\mathfrak{F}: (\mathcal{F} \cap \bigcup^{\mathfrak{F}} S \neq \emptyset \Rightarrow \exists \mathcal{K} \in S: \mathcal{F} \cap \mathcal{K} \neq \emptyset)$

$\forall S \in \mathscr{P}\mathfrak{F}: (\langle f \rangle A \cap \bigcup^{\mathfrak{F}} S \neq \emptyset \Rightarrow \exists \mathcal{K} \in S: \langle f \rangle A \cap \mathcal{K} \neq \emptyset)$

$\forall S \in \mathscr{P}\mathfrak{F}: (A\, [f] \bigcup^{\mathfrak{F}} S \Rightarrow \exists \mathcal{K} \in S: A\, [f]\, \mathcal{K})$

$\forall S \in \mathscr{P}\mathfrak{F}: \langle f \rangle \bigcup^{\mathfrak{F}} S = \bigcup^{\mathfrak{F}} \langle \langle f \rangle \rangle S$;

$\forall S \in \mathscr{P}\mathscr{P}\mho: \langle f \rangle \bigcup S = \bigcup^{\mathfrak{F}} \langle \langle f \rangle \rangle S$

Proposition 52. *Equivalence of morphisms is an equivalence relation.*

Proof.

Reflexity. Follows from the identity.

Symmetry. Obvious.

Transitivity. Let $f \sim g$ and $g \sim h$. Then there exist a morphism p such that $\operatorname{Src} p \sqsubseteq \operatorname{Src} f$, $\operatorname{Src} p \sqsubseteq \operatorname{Src} g$, $\operatorname{Dst} p \sqsubseteq \operatorname{Dst} f$, $\operatorname{Dst} p \sqsubseteq \operatorname{Dst} g$ and $\iota_{\operatorname{Src} f, \operatorname{Dst} f}\, p = f$ and $\iota_{\operatorname{Src} g, \operatorname{Dst} g}\, p = g$ and ??

??

$f = \iota_{\operatorname{Src} f, \operatorname{Dst} f}\, p = (\operatorname{Dst} p \hookrightarrow \operatorname{Dst} f) \circ p \circ (\operatorname{Src} p \hookrightarrow \operatorname{Src} f)^{\dagger}$

$g = \iota_{\operatorname{Src} g, \operatorname{Dst} g}\, p = (\operatorname{Dst} p \hookrightarrow \operatorname{Dst} g) \circ p \circ (\operatorname{Src} p \hookrightarrow \operatorname{Src} g)^{\dagger}$

$g = \iota_{\operatorname{Src} g, \operatorname{Dst} g}\, q = (\operatorname{Dst} q \hookrightarrow \operatorname{Dst} g) \circ q \circ (\operatorname{Src} q \hookrightarrow \operatorname{Src} g)^{\dagger}$

$h = \iota_{\operatorname{Src} h, \operatorname{Dst} h}\, q = (\operatorname{Dst} q \hookrightarrow \operatorname{Dst} h) \circ q \circ (\operatorname{Src} q \hookrightarrow \operatorname{Src} h)^{\dagger}$

??

We have like:

$(X \hookrightarrow A) \circ p = (Y \hookrightarrow A) \circ q$ and need $z \sqsubseteq p, q$ such that

$$(\operatorname{Dst} z \hookrightarrow A) \circ z = (X \hookrightarrow A) \circ p = (Y \hookrightarrow A) \circ q.$$

Take $z = p \sqcap q$. Repeating this, we get:

$g = \iota_{\operatorname{Src} g, \operatorname{Dst} g}\, p = (\operatorname{Dst} z \hookrightarrow \operatorname{Dst} g) \circ z \circ (\operatorname{Src} z \hookrightarrow \operatorname{Src} g)^{\dagger}$

$g = \iota_{\operatorname{Src} g, \operatorname{Dst} g}\, q = (\operatorname{Dst} z \hookrightarrow \operatorname{Dst} g) \circ z \circ (\operatorname{Src} z \hookrightarrow \operatorname{Src} g)^{\dagger}$

??

Axiom: $X \hookrightarrow A$ is metamonovalued (requires order on the set of **all** objects). □

Conjecture 53. $(\mathsf{RLD})_{\mathrm{in}} f = \bigsqcap^{\mathsf{RLD}} \mathrm{up}^{\Gamma(\operatorname{Src} f; \operatorname{Dst} f)} f$ *for every funcoid* f.

Proof. Let $K \in (\mathsf{RLD})_{\mathrm{in}} f$.

$\bigsqcap^{\mathsf{RLD}} \mathrm{up}^{\Gamma(\operatorname{Src} f; \operatorname{Dst} f)} f = \bigsqcap^{\mathsf{RLD}} \mathrm{up}^{\Gamma(\operatorname{Src} f; \operatorname{Dst} f)} \bigsqcup \operatorname{atoms} f \sqsubseteq \bigsqcap^{\mathsf{RLD}} \mathrm{up}^{\Gamma(\operatorname{Src} f; \operatorname{Dst} f)} \bigsqcup^{\mathsf{FCD}} \{X_{\mathcal{A}} \times Y_{\mathcal{B}} \mid \mathcal{A} \in \mathfrak{F}(\operatorname{Src} f), \mathcal{B} \in \mathfrak{F}(\operatorname{Dst} f), \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \sqsubseteq f\} = \bigsqcap^{\mathsf{RLD}} \mathrm{up}^{\Gamma(\operatorname{Src} f; \operatorname{Dst} f)} \bigcup \{X_{\mathcal{A}} \times Y_{\mathcal{B}} \mid \mathcal{A} \in \mathfrak{F}(\operatorname{Src} f), \mathcal{B} \in \mathfrak{F}(\operatorname{Dst} f), \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \sqsubseteq f\}$

Pattern theorem 8.19. □

Conjecture 54. $(\mathsf{FCD}) f = \bigsqcap^{\mathsf{FCD}} (\Gamma(A; B) \cap \operatorname{GR} f)$ *for every reloid* $f \in \mathsf{RLD}(A; B)$.

Proof. $x \times^{\mathsf{FCD}} y \in \operatorname{atoms} (\mathsf{FCD}) f \Leftrightarrow x \times^{\mathsf{RLD}} y \not\asymp f \Leftrightarrow$??