$\operatorname{im}(\mathsf{RLD})_{\mathrm{in}} f = \operatorname{im} f$ is similar. □

Theorem 41. *A reloid f is monovalued iff $\forall g \in \mathsf{RLD}: (g \subseteq f \Rightarrow g = f|^{\mathsf{RLD}}_{\operatorname{dom} g})$.*

Proof.

⇒. Let f is monovalued. Then exists $F \in \operatorname{up} f$ such that $f = F|^{\mathsf{RLD}}_{\operatorname{dom} f}$. Let $g \in \mathsf{RLD}$ and $g \subseteq f$. Then exists $G \in \operatorname{up} g$ such that $g = G|^{\mathsf{RLD}}_{\operatorname{dom} g}$. We have $g = g \cap^{\mathsf{RLD}} f = G|^{\mathsf{RLD}}_{\operatorname{dom} g} \cap^{\mathsf{RLD}} F|^{\mathsf{RLD}}_{\operatorname{dom} f} = G|^{\mathsf{RLD}}_{\operatorname{dom} g} \cap^{\mathsf{RLD}} F|^{\mathsf{RLD}}_{\operatorname{dom} g} = (G \cap^{\mathsf{RLD}} F)|^{\mathsf{RLD}}_{\operatorname{dom} g} \supseteq f|^{\mathsf{RLD}}_{\operatorname{dom} g}$. But obviously $g \subseteq f|^{\mathsf{RLD}}_{\operatorname{dom} g}$. So $g = f|^{\mathsf{RLD}}_{\operatorname{dom} g}$. □

Conjecture 42. $\operatorname{Compl} f = f \setminus^{*\,\mathsf{FCD}} (\Omega \times^{\mathsf{FCD}} \mho)$ *for every funcoid f.*

This conjecture may be proved by considerations similar to these in the section "Fréchet filter" in [?].

Example 43. $(\mathsf{RLD})_{\mathrm{in}}$ is not a lower adjoint (in general).

Proof. Enough to prove one of the following:

Enough to prove non-existence of $\max\{f \in \mathsf{FCD} \mid (\mathsf{RLD})_{\mathrm{in}} f \subseteq g\}$ for some reloid g.

??Enough to prove $(\mathsf{RLD})_{\mathrm{in}} \bigcup^{\mathsf{FCD}} S \neq \bigcup^{\mathsf{FCD}} \langle (\mathsf{RLD})_{\mathrm{in}} \rangle S$ for some set S of funcoids.

?? □

Theorem 44. *A filter $\mathcal{A}$ is connected regarding a reloid f iff it is connected regarding the funcoid $(\mathsf{FCD}) f$.*

Proof. $\mathcal{A}$ is connected regarding f iff $\mathcal{A}$ is connected regarding every element of $F \in \operatorname{up} f$ (considered as reloids) that is iff $S^*(F \cap^{\mathsf{RLD}} (\mathcal{A} \times^{\mathsf{RLD}} \mathcal{A})) = \mathcal{A} \times^{\mathsf{RLD}} \mathcal{A}$.

?? □

Theorem 45. $(\mathsf{RLD})_{\mathrm{out}} (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}) \neq \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$ *in general.*

Proof. $(\mathsf{RLD})_{\mathrm{out}} (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}) = \bigcap^{\mathsf{RLD}} \operatorname{up}(\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B})$

If the equality holds, $\forall F \in \operatorname{up}(\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}) \exists A \in \operatorname{up} \mathcal{A}, B \in \operatorname{up} \mathcal{B}: A \times B \subseteq F$.

Let $\mathcal{L}$, $\mathcal{B}$ are f.o. and the cardinality of the set $H = \{\mathcal{Y} \in \mathfrak{F} \mid \mathcal{B} \subset \mathcal{Y} \subseteq \mathcal{L}\}$ is infinite but not greater than cardinality of a set A. (We can always assume existence of such set A, extending the base set $\mho$ above if necessary.)

For every f.o. $\mathcal{Y} \in H$ choose a set $Y_{\mathcal{Y}} \in \operatorname{up} \mathcal{B}$ such that $Y_{\mathcal{Y}} \notin \operatorname{up} \mathcal{Y}$

Let z is a surjection from A to H.

Consider the binary relation $F = \bigcup \{\{\alpha\} \times Y_{z\alpha} \mid \alpha \in A\}$.

We have $F \supseteq A \times^{\mathsf{FCD}} \mathcal{B}$ that is $F \in \operatorname{up}(\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B})$.

But if $B \in \operatorname{up} \mathcal{B}$ then ?? $A \times B \nsubseteq F$ because $F\alpha = Y_{z\alpha} \notin \operatorname{up} z\alpha$ that is $\forall \mathcal{Y} \supset \mathcal{B}: Y_{z\alpha} \notin \operatorname{up} \mathcal{Y}$

[Not needed] Consider funcoid $f = \bigcup^{\mathsf{FCD}} \{\{\alpha\} \times^{\mathsf{FCD}} z\alpha \mid \alpha \in A\}$.

?? □

Theorem 46. $\bigcap^{Z(D\mathcal{A})} \{X \cap^{\mathfrak{F}} \mathcal{A} \mid X \in T\} = \mathcal{A} \cap^{\mathfrak{F}} \bigcap T$.

Proof. That $\mathcal{A} \cap^{\mathfrak{F}} \bigcap T$ is a lower bound of $\{X \cap^{\mathfrak{F}} \mathcal{A} \mid X \in T\}$ is obvious.

We need to prove that it is the greatest lower bound, that is that for every lower bound $\mathcal{B} \in Z(D\mathcal{A})$ of $\{X \cap^{\mathfrak{F}} \mathcal{A} \mid X \in T\}$ we have $\mathcal{A} \cap^{\mathfrak{F}} \bigcap T \supseteq \mathcal{B}$.

Let $\mathcal{B} = B \cap^{\mathfrak{F}} \mathcal{A}$ is a lower bound of $\{X \cap^{\mathfrak{F}} \mathcal{A} \mid X \in T\}$ that is $\forall X \in T: B \cap^{\mathfrak{F}} \mathcal{A} \subseteq X \cap^{\mathfrak{F}} \mathcal{A}$. Left to prove that $\mathcal{A} \cap^{\mathfrak{F}} \bigcap T \supseteq B \cap^{\mathfrak{F}} \mathcal{A}$.

$(X \cup B) \cap^{\mathfrak{F}} \mathcal{A} = (X \cap^{\mathfrak{F}} \mathcal{A}) \cup^{\mathfrak{F}} (B \cap^{\mathfrak{F}} \mathcal{A}) = X \cap^{\mathfrak{F}} \mathcal{A}$

$B \cup^{\mathfrak{F}} (\mathcal{A} \cap^{\mathfrak{F}} \bigcap T) = (B \cup^{\mathfrak{F}} \mathcal{A}) \cap^{\mathfrak{F}} (B \cup \bigcap T)$