Theorem 36. *If f is a complete reloids and S is a set of complete reloids. Then [TODO: The same for funcoids?]*

$$f \cap^{\mathsf{RLD}} \bigcup S = \bigcup \langle f \cap^{\mathsf{RLD}} \rangle S.$$

Theorem 37. *Composition with a (co?)complete reloid is an adjoint:*

Proof. $F \circ$ is a lower adjoint. Let ξ is its upper adjoint. Then

$F \circ x \subseteq y \Leftrightarrow x \subseteq \xi(y)$

$x \subseteq \xi(F \circ x) \Leftrightarrow F \circ \xi(y)$

$\xi(b) = \max \{x \in \mathsf{RLD} \mid F \circ x \subseteq b\}$ (a proof circle follows this)

$x \subseteq F^{-1} \circ b \Rightarrow F \circ x \subseteq F \circ b$

$F \circ \bigcup^{\mathsf{RLD}} R = \bigcap^{\mathsf{RLD}} \{F \circ K \mid K \in \mathrm{up} \bigcup^{\mathsf{RLD}} R\}$

$\bigcup^{\mathsf{RLD}} \langle F \circ \rangle R = \bigcup^{\mathsf{RLD}} \{\bigcap^{\mathsf{RLD}} \{F \circ G \mid G \in \mathrm{up}\, g\} \mid g \in R\}$ □

Conjecture 38. $\operatorname{Compl} f \cap^{\mathsf{RLD}} \operatorname{Compl} g = \operatorname{Compl}(f \cap^{\mathsf{RLD}} g)$ for every reloids f and g.

Proof. $\operatorname{Compl}(f \cap^{\mathsf{RLD}} g) = \bigcup^{\mathsf{RLD}} \{(f \cap^{\mathsf{RLD}} g)|^{\mathsf{RLD}}_{\{\alpha\}} \mid \alpha \in \mho\}$

$\operatorname{Compl} f \cap^{\mathsf{RLD}} \operatorname{Compl} g = \bigcup^{\mathsf{RLD}} \{f|^{\mathsf{RLD}}_{\{\alpha\}} \mid \alpha \in \mho\} \cap^{\mathsf{RLD}} \bigcup^{\mathsf{RLD}} \{g|^{\mathsf{RLD}}_{\{\alpha\}} \mid \alpha \in \mho\}$

$(\operatorname{Compl}(f \cap^{\mathsf{RLD}} g))|^{\mathsf{RLD}}_{\{\beta\}} = (f \cap^{\mathsf{RLD}} g)|^{\mathsf{RLD}}_{\{\beta\}}$

$(\operatorname{Compl} f \cap^{\mathsf{RLD}} \operatorname{Compl} g)|^{\mathsf{RLD}}_{\{\beta\}} = (f \cap^{\mathsf{RLD}} g)|^{\mathsf{RLD}}_{\{\beta\}}$.

So enough to prove that $\operatorname{Compl} f \cap^{\mathsf{RLD}} \operatorname{Compl} g$ is complete.

Let $A = \operatorname{atoms}^{\mathsf{ComplRLD}} f$ and $B = \operatorname{atoms}^{\mathsf{ComplRLD}} g$. Then ??

Obviously $\operatorname{Compl} f \cap^{\mathsf{RLD}} \operatorname{Compl} g \supseteq \operatorname{Compl} f \cap^{\mathsf{ComplRLD}} \operatorname{Compl} g$

Suppose it exists $a \in \operatorname{atoms}^{\mathsf{RLD}}(\operatorname{Compl} f \cap^{\mathsf{RLD}} \operatorname{Compl} g)$ such that $a \notin$ $\operatorname{toms}^{\mathsf{RLD}}(\operatorname{Compl} f \cap^{\mathsf{ComplRLD}} \operatorname{Compl} g)$. Then ?? □

Conjecture 39. *If f and g are reloids, then*

$$g \circ f = \bigcup^{\mathsf{RLD}} \{G \circ F \mid F \in \operatorname{atoms}^{\mathsf{RLD}} f, G \in \operatorname{atoms}^{\mathsf{RLD}} g\}.$$

Proof. $g \circ f = ?? = g \circ \bigcup^{\mathsf{RLD}} \operatorname{atoms}^{\mathsf{RLD}} f = ?? = \bigcup^{\mathsf{RLD}} \langle g \circ \rangle \operatorname{atoms}^{\mathsf{RLD}} f$

$\bigcup^{\mathsf{RLD}} \{G \circ F \mid F \in \operatorname{atoms}^{\mathsf{RLD}} f, G \in \operatorname{atoms}^{\mathsf{RLD}} g\} s \subseteq \bigcup^{\mathsf{RLD}} \{g \circ F \mid F \in \operatorname{atoms}^{\mathsf{RLD}} f\} \subseteq g \circ f$

?? □

Theorem 40. $\operatorname{dom} (\mathsf{RLD})_{\mathrm{in}} f = \operatorname{dom} f$ *and* $\operatorname{im} (\mathsf{RLD})_{\mathrm{in}} f = \operatorname{im} f$ *for every funcoid* f.

Proof. Let an atomic f.o. $a \subseteq \operatorname{dom} f$. Then exists atomic f.o. $b \subseteq \operatorname{im} f$ such that $a \times^{\mathsf{FCD}} b \subseteq f$. Consequently

$a \times^{\mathsf{RLD}} b \subseteq (\mathsf{RLD})_{\mathrm{in}} f \Rightarrow \forall K \in \mathrm{up}\, (\mathsf{RLD})_{\mathrm{in}} f: a \times^{\mathsf{RLD}} b \subseteq K \Rightarrow \forall K \in \mathrm{up}\, (\mathsf{RLD})_{\mathrm{in}} f: a \subseteq \operatorname{dom} K \Leftrightarrow$ $a \subseteq \bigcap^{\mathfrak{F}} \langle \operatorname{dom} \rangle \mathrm{up}\, (\mathsf{RLD})_{\mathrm{in}} f \Leftrightarrow a \subseteq \operatorname{dom} (\mathsf{RLD})_{\mathrm{in}} f$.

Let now an atomic f.o. $a \subseteq \operatorname{dom} (\mathsf{RLD})_{\mathrm{in}} f$. Then $\forall K \in \mathrm{up}\, (\mathsf{RLD})_{\mathrm{in}} f: a \subseteq \operatorname{dom} K$

What is equivalent to

$$\forall K \in \bigcap \{\mathrm{up}(a \times^{\mathsf{RLD}} b) \mid a, b \in \operatorname{atoms}^{\mathfrak{F}} \mho, a \times^{\mathsf{FCD}} b \subseteq f\}: a \subseteq \operatorname{dom} K$$

Let $K \in \mathrm{up}\, f$. Then $K \supseteq a \times^{\mathsf{FCD}} b$ for every $a, b \in \operatorname{atoms}^{\mathfrak{F}} \mho$ where $a \times^{\mathsf{FCD}} b \subseteq f$ that is exist ?? $K \in \mathrm{up}(a \times^{\mathsf{RLD}} b)$ for ??

??

from what follows?? [FIXME: b is depended on K] that exist $b \subseteq \operatorname{im} f$ such that $\forall K \in \mathrm{up}\, (\mathsf{RLD})_{\mathrm{in}} f: a \times^{\mathsf{RLD}} b \subseteq K$ that is $\forall K \in \mathrm{up}\, (\mathsf{RLD})_{\mathrm{in}} f: K \in \mathrm{up}(a \times^{\mathsf{RLD}} b)$ and thus $(\mathsf{RLD})_{\mathrm{in}} f \supseteq a \times^{\mathsf{RLD}} b$ and consequently $\operatorname{dom} (\mathsf{RLD})_{\mathrm{in}} f \supseteq \operatorname{dom}(a \times^{\mathsf{RLD}} b) = a$.

Thus $a \subseteq \operatorname{dom} f \Leftrightarrow a \subseteq \operatorname{dom} (\mathsf{RLD})_{\mathrm{in}} f$ for each atomic f.o. a from what follows $\operatorname{dom} (\mathsf{RLD})_{\mathrm{in}} f = \operatorname{dom} f$.