If $X\in\{\{0\}\}$, $Y\in\{\{1\}\}$ then $X\sqcup Y\notin\{\{0\},\{0,1\}\}$

??

That $X_i=\emptyset\Rightarrow X\notin(\operatorname{val} f)_k L$ is obvious. So f is a pre-multifuncoid.

?? □

Conjecture 29. *If a is a completary multifuncoid and $\operatorname{Dst} f_i$ is a starrish poset for every $i\in n$ then $\operatorname{StarComp}(a;f)$ is a completary multifuncoid.*

Proof. Let $\forall K\in\prod \operatorname{form} f\colon (K\sqsupseteq L_0\wedge K\sqsupseteq L_1\Rightarrow K\in\operatorname{StarComp}(a;f))$ that is $\forall K\in\prod\operatorname{form} f\colon \left(K\sqsupseteq L_0\wedge K\sqsupseteq L_1\Rightarrow\exists y\in\prod_{i\in n}\operatorname{atoms}\mathfrak{A}_i\colon(\forall i\in n\colon y_i\,[f_i]\,K_i\wedge y\in a)\right)$ that is $\forall K\in\prod\operatorname{form} f\exists y\in\prod_{i\in n}\operatorname{atoms}\mathfrak{A}_i\colon(K\sqsupseteq L_0\wedge K\sqsupseteq L_1\Rightarrow(\forall i\in n\colon y_i\,[f_i]\,K_i\wedge y\in a))$ that is $\forall K\in\prod\operatorname{form} f\exists y\in\prod_{i\in n}\operatorname{atoms}\mathfrak{A}_i\colon((K\sqsupseteq L_0\wedge K\sqsupseteq L_1\Rightarrow\forall i\in n\colon y_i\,[f_i]\,K_i)\wedge y\in a)$ that is?? $\forall K\in\prod\operatorname{form} f\exists y\in\prod_{i\in n}\operatorname{atoms}\mathfrak{A}_i\colon(\exists c\in\{0,1\}^n\forall i\in n\colon y_i\,[f_i]\,L_{c(i)}\,i\wedge y\in a)$ that is ?? □

Conjecture 30. *$\prod^{(D)}F$ is a pre-multifuncoid if every F_i is a pre-multifuncoid.*

Proof. Let $X,Y\in\left(\operatorname{form}\prod^{(D)}F\right)_{(i;j)}$.

$\forall Z\in\left(\operatorname{form}\prod^{(D)}F\right)_{(i;j)}\colon\left(Z\supseteq X\wedge Z\supseteq Y\Rightarrow Z\in\left(\operatorname{val}\prod^{(D)}F\right)_{(i;j)}L\right)\Leftrightarrow\forall Z\in\left(\operatorname{form}\prod^{(D)}F\right)_{(i;j)}\colon(Z\supseteq X\wedge Z\supseteq Y\Rightarrow\exists K\in(\operatorname{form}F_i)|_{(\operatorname{arity}F_i)\setminus\{j\}}\colon Z\in(\operatorname{val}F_j)K)\Leftrightarrow\forall Z\in\left(\operatorname{form}\prod^{(D)}F\right)_{(i;j)}\colon(\exists K\in(\operatorname{form}F_i)|_{(\operatorname{arity}F_i)\setminus\{j\}}\colon(Z\supseteq X\wedge Z\supseteq Y\Rightarrow Z\in(\operatorname{val}F_j)K))\Leftrightarrow$?? $\Leftrightarrow\forall Z\in\left(\operatorname{form}\prod^{(D)}F\right)_{(i;j)}\colon(\exists K\in(\operatorname{form}F_i)|_{(\operatorname{arity}F_i)\setminus\{j\}}\colon(X\in(\operatorname{val}F_j)K\vee Y\in(\operatorname{val}F_j)K))\Leftrightarrow\forall Z\in\left(\operatorname{form}\prod^{(D)}F\right)_{(i;j)}\colon(\exists K\in(\operatorname{form}F_i)|_{(\operatorname{arity}F_i)\setminus\{j\}}\colon X\in(\operatorname{val}F_j)K\vee\exists K\in(\operatorname{form}F_i)|_{(\operatorname{arity}F_i)\setminus\{j\}}\colon Y\in(\operatorname{val}F_j)K)\Leftrightarrow$ □

Let f is a funcoid.

Then there exists a reloid g such that ??

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$(\mathsf{RLD})_{\mathrm{in}}f=\bigcup\left\{a\times^{\mathsf{RLD}}b\mid a\in\operatorname{atoms}1^{\mathfrak{F}(\operatorname{Src}f)},b\in\operatorname{atoms}1^{\mathfrak{F}(\operatorname{Dst}f)},a\times^{\mathsf{FCD}}b\subseteq f\right\}$

$(\mathsf{RLD})_{\mathrm{in}}(g\circ f)=\bigcup\left\{a\times^{\mathsf{RLD}}b\mid a\in\operatorname{atoms}1^{\mathfrak{F}(\operatorname{Src}f)},b\in\operatorname{atoms}1^{\mathfrak{F}(\operatorname{Dst}f)},a\times^{\mathsf{FCD}}b\subseteq g\circ f\right\}=\bigcup\left\{a\times^{\mathsf{RLD}}b\mid a\in\operatorname{atoms}1^{\mathfrak{F}(\operatorname{Src}f)},b\in\operatorname{atoms}1^{\mathfrak{F}(\operatorname{Dst}f)},a\times^{\mathsf{RLD}}b\subseteq(\mathsf{RLD})_{\mathrm{in}}(g\circ f)\right\}$

$(\mathsf{RLD})_{\mathrm{in}}(\mathsf{FCD})((\mathsf{RLD})_{\mathrm{in}}g\circ(\mathsf{RLD})_{\mathrm{in}}f)=(\mathsf{RLD})_{\mathrm{in}}((\mathsf{FCD})(\mathsf{RLD})_{\mathrm{in}}g\circ(\mathsf{FCD})(\mathsf{RLD})_{\mathrm{in}}f)=(\mathsf{RLD})_{\mathrm{in}}(g\circ f)$

Lemma 31. $\forall Y\in\operatorname{up}\langle f\rangle^*X\exists F\in\operatorname{up}f\colon\langle F\rangle X\subseteq Y$ *for every funcoid f.*

Proof. ?? □

$(\mathsf{RLD})_{\mathrm{in}}(g\circ f)=$

Theorem 32. $g\circ f=\bigcap\left\{\uparrow^{\mathsf{FCD}(\operatorname{Src}f;\operatorname{Dst}g)}(G\circ F)\mid F\in\operatorname{up}f,G\in\operatorname{up}g\right\}$

Proof. It's enough?? to prove that $\forall H\in\operatorname{up}(g\circ f)\exists F\in\operatorname{up}f,G\in\operatorname{up}g\colon H\supseteq G\circ F$.

$X\,[g\circ f]^*\,Y\Leftrightarrow\langle f\rangle^*X\not\asymp\langle g^{-1}\rangle^*Y\Leftrightarrow\forall X'\in\operatorname{up}\langle f\rangle^*X,Y'\in\operatorname{up}\langle g^{-1}\rangle^*Y\colon X'\not\asymp Y'\Leftrightarrow\exists F\in\operatorname{up}f,G\in\operatorname{up}g\colon\langle F\rangle X\not\asymp\langle G\rangle Y\Leftrightarrow\exists F\in\operatorname{up}f,G\in\operatorname{up}g\colon X\,[G\circ F]\,Y$ (used the lemma).

Let $H\in\operatorname{up}(g\circ f)$. Then $X\,[H]^*\,Y\Rightarrow X\,[g\circ f]^*\,Y\Rightarrow\exists F\in\operatorname{up}f,G\in\operatorname{up}g\colon X\,[G\circ F]\,Y$ for every X, Y. Thus ??(it does not work because F amd G depend on X and Y). □

Lemma 33. $f\circ r=\bigcap\{f\circ\uparrow^{\mathsf{FCD}}R\mid R\in\operatorname{up}r\}$

Proof. Obviously $f\circ r\subseteq\bigcap\{f\circ\uparrow^{\mathsf{FCD}}R\mid R\in\operatorname{up}r\}$.

$\langle f\circ r\rangle^*X\supseteq$??$\bigcap\{\langle f\rangle\langle R\rangle X\mid R\in\operatorname{up}r\}=\bigcap\{\langle f\rangle\langle\uparrow^{\mathsf{FCD}}R\rangle^*X\mid R\in\operatorname{up}r\}=\bigcap\{\langle f\circ\uparrow^{\mathsf{FCD}}R\rangle^*X\mid R\in\operatorname{up}r\}\supseteq\langle\bigcap\{f\circ\uparrow^{\mathsf{FCD}}R\mid R\in\operatorname{up}r\}\rangle^*X$