6 Rest

Lemma 24. *Every non-empty set has a well ordering with greatest element.*

Proof. Take an arbitrary well ordering of our set and move the first element to the end of the order. □

Theorem 25. $L\in[f]\Rightarrow[f]\cap\prod_{i\in\mathrm{dom}\,\mathfrak{A}}\mathrm{atoms}\,L_i\neq\emptyset$ *for every pre-multifuncoid* f *of the form whose elements are atomic posets.*

Proof. If $\mathrm{arity}\,f=0$ our theorem is trivial, so let $\mathrm{arity}\,f\neq0$. Let $\sqsubseteq$ is a well-ordering of $\mathrm{arity}\,f$ with greatest element m (it exists by the lemma).

Let Φ is a function which maps non-least elements of posets into atoms under these elements and least elements into themselves. (Note that Φ is defined on least elements only for completeness, Φ is never taken on a least element in the proof below.) [TODO: Fix the "universal set" paradox here.]

Define a transfinite sequence a by transfinite induction with the formula

$$a_c=\Phi\langle f\rangle_c\,(a|_{X(c)\setminus\{c\}}\cup L|_{(\mathrm{arity}\,f)\setminus X(c)}).$$

Let $b_c=a|_{X(c)\setminus\{c\}}\cup L|_{(\mathrm{arity}\,f)\setminus X(c)}$. Then $a_c=\Phi\langle f\rangle_c\,b_c$.

Let us prove by transfinite induction

$$a_c\in\mathrm{atoms}\,L_c.$$

$a_c=\Phi\langle f\rangle_c\,L|_{(\mathrm{arity}\,f)\setminus\{c\}}\sqsubseteq\langle f\rangle_c\,L|_{(\mathrm{arity}\,f)\setminus\{c\}}$. Thus $a_c\sqsubseteq L_c$. [TODO: Is it true for *pre*-multifuncoids?]

The only thing remained to prove is that $\langle f\rangle_c\,b_c\neq0$

that is $\langle f\rangle_c\,(a|_{X(c)\setminus\{c\}}\cup L|_{(\mathrm{arity}\,f)\setminus X(c)})\neq0$ that is $y\not\asymp\langle f\rangle_c\,b_c$

??

$L_c\not\asymp\langle f\rangle_c\;b_c\Leftrightarrow b_c(0)\not\asymp\langle f\rangle_0(b_c|_{(\mathrm{arity}\,f)\setminus\{0\}}\cup\{c;\;L_c\})\Leftrightarrow a_0\not\asymp\langle f\rangle_0(a|_{X(c)\setminus\{0,c\}}\cup L|_{(\mathrm{arity}\,f)\setminus X(c)}\cup\{c;L_c\})\Leftrightarrow a_0\not\asymp\langle f\rangle_0(a|_{X(c)\setminus\{0,c\}}\cup L|_{((\mathrm{arity}\,f)\setminus X(c))\cup\{c\}})\Leftarrow a_0\not\asymp\langle f\rangle_0(a|_{X(c)\setminus\{0,c\}}\cup a|_{((\mathrm{arity}\,f)\setminus X(c))\cup\{c\}})\Leftrightarrow a_0\not\asymp\langle f\rangle_0(a|_{(X(c)\setminus\{0,c\})\cup((\mathrm{arity}\,f)\setminus X(c))\cup\{c\}})\Leftrightarrow a_0\not\asymp\langle f\rangle_0(a|_{(\mathrm{arity}\,f)\setminus\{0\}})\Leftrightarrow\Phi\langle f\rangle_0\,(L|_{(\mathrm{arity}\,f)\setminus\{0\}})\not\asymp\langle f\rangle_0(a|_{(\mathrm{arity}\,f)\setminus\{0\}})$.

??

$a_0=\Phi\langle f\rangle_0\,(L|_{(\mathrm{arity}\,f)\setminus\{0\}})$

??

??Two ways to prove:??

$L_c\not\asymp\langle f\rangle_c\;b_c\Leftrightarrow b_c(k)\not\asymp\langle f\rangle_k(b_c|_{(\mathrm{arity}\,f)\setminus\{k\}}\cup\{c;\;L_c\})\Leftrightarrow a_k\not\asymp\langle f\rangle_k(a|_{X(c)\setminus\{k,c\}}\cup L|_{(\mathrm{arity}\,f)\setminus X(c)}\cup\{c;L_c\})\Leftrightarrow a_k\not\asymp\langle f\rangle_k(a|_{X(c)\setminus\{k,c\}}\cup L|_{((\mathrm{arity}\,f)\setminus X(c))\cup\{c\}})\Leftarrow a_k\not\asymp\langle f\rangle_k(a|_{X(c)\setminus\{k,c\}}\cup a|_{((\mathrm{arity}\,f)\setminus X(c))\cup\{c\}})\Leftrightarrow a_k\not\asymp\langle f\rangle_k(a|_{(X(c)\setminus\{k,c\})\cup((\mathrm{arity}\,f)\setminus X(c))\cup\{c\}})\Leftrightarrow a_k\not\asymp\langle f\rangle_k(a|_{(\mathrm{arity}\,f)\setminus\{k\}})$

??

$L_c\not\asymp\langle f\rangle_c\,b_c\Leftrightarrow a_k\not\asymp\langle f\rangle_k\,(a|_{X(c)\setminus\{c,k\}}\cup L|_{(\mathrm{arity}\,f)\setminus X(c)}\cup\{(c;\,L_c)\})\Leftrightarrow a_k\not\asymp\langle f\rangle_k\,(a|_{X(c)\setminus\{c,k\}}\cup L|_{((\mathrm{arity}\,f)\setminus X(c))\cup\{c\}})\Leftarrow a_k\not\asymp\langle f\rangle_k\,(a|_{X(c)\setminus\{c,k\}}\cup a|_{((\mathrm{arity}\,f)\setminus X(c))\cup\{c\}})\Leftrightarrow a_k\not\asymp\langle f\rangle_k\,(a|_{(\mathrm{arity}\,f)\setminus\{k\}})$

??

$y\not\asymp\langle f\rangle_c\;b_c\Leftrightarrow a_k\not\asymp\langle f\rangle_k\;(a|_{X(c)\setminus\{c,k\}}\cup L|_{(\mathrm{arity}\,f)\setminus X(c)}\cup\{(c;\;y)\})\Leftarrow a_k\not\asymp\langle f\rangle_k\;(a|_{X(k)\setminus\{k\}}\cup L|_{(\mathrm{arity}\,f)\setminus X(c)}\cup\{(c;y)\})$

$y\not\asymp\langle f\rangle_c\,b_c\Leftrightarrow L_k\not\asymp\langle f\rangle_k\,(a|_{X(c)\setminus\{c\}}\cup L|_{(\mathrm{arity}\,f)\setminus(X(c)\cup\{k\})}\cup\{(c;y)\})\Leftarrow$

??

$a_m=\Phi\langle f\rangle_m(\lambda i\in(\mathrm{arity}\;f)\setminus\{m\}\colon a_i)=\Phi\langle f\rangle_m\;a|_{(\mathrm{arity}\,f)\setminus\{m\}}$; $a_m\not\asymp\Phi\langle f\rangle_m\;a|_{(\mathrm{arity}\,f)\setminus\{m\}}$; $a_m\not\asymp\langle f\rangle_m\,a|_{(\mathrm{arity}\,f)\setminus\{m\}}$; $a\in[f]$. □

Theorem 26. *Let* $\mathfrak{A}=\mathfrak{A}_{i\in n}$ *is a family of boolean lattices.*

A relation $\delta\in\mathscr{P}\prod\mathrm{atoms}^{\mathfrak{F}(\mathfrak{A}_i)}$ *such that for every* $a\in\prod\mathrm{atoms}^{\mathfrak{F}(\mathfrak{A}_i)}$

$$\forall A\in a\colon\delta\cap\prod_{i\in n}\mathrm{atoms}\uparrow^{\mathfrak{A}_i}A_i\neq\emptyset\Rightarrow a\in\delta\qquad(3)$$

can be continued till the function $\Uparrow f$ *for a unique staroid* f *of the form* $\lambda i\in n\colon\mathfrak{P}(\mathfrak{A}_i)$. *The funcoid* f *is completary.*