4 Exponentials in category Rld

If now G, H are endoreloids, then $\operatorname{MOR}(G;H)$ is an endoreloid.

Definition 17. $\operatorname{MOR}_\alpha(G;H)=\bigsqcup\left\{t\in\operatorname{atoms}^{\mathsf{RLD}((\operatorname{Ob}H)^{\operatorname{Ob}G};(\operatorname{Ob}H)^{\operatorname{Ob}G})}\mid(\operatorname{im}t)\circ G\circ(\operatorname{dom}t)^{-1}\sqsubseteq H\right\}$.

Proposition 18. $\operatorname{MOR}_\alpha(G;H)=\bigsqcup\left\{t_0\times^{\mathsf{RLD}}t_1\mid t_0,t_1\in\operatorname{atoms}^{\mathfrak{F}((\operatorname{Ob}H)^{\operatorname{Ob}G})},t_1\circ G\circ t_0{}^{-1}\sqsubseteq H\right\}$.

Proof. If $t_0,t_1\in\mathfrak{F}((\operatorname{Ob}H)^{\operatorname{Ob}G})$ and $t_1\circ G\circ t_0{}^{-1}\sqsubseteq H$ then there are $t\in\operatorname{atoms}(t_0\times^{\mathsf{RLD}}t_1)$. Thus $(\operatorname{im}t)\circ G\circ(\operatorname{dom}t)^{-1}\sqsubseteq H$ and $t\in\operatorname{atoms}^{\mathsf{RLD}((\operatorname{Ob}H)^{\operatorname{Ob}G};(\operatorname{Ob}H)^{\operatorname{Ob}G})}$.

If $t\in\operatorname{atoms}^{\mathsf{RLD}((\operatorname{Ob}H)^{\operatorname{Ob}G};(\operatorname{Ob}H)^{\operatorname{Ob}G})}$ and $(\operatorname{im}t)\circ G\circ(\operatorname{dom}t)^{-1}\sqsubseteq H$ then $\operatorname{dom}t=t_0$ and $\operatorname{im}t=t_1$ for some $t_0,t_1\in\operatorname{atoms}^{\mathfrak{F}((\operatorname{Ob}H)^{\operatorname{Ob}G})}$ and thus $t_1\circ G\circ t_0{}^{-1}\sqsubseteq H$. □

Definition 19. $\operatorname{MOR}_\beta(G;H)=\bigsqcup\{t\in\mathsf{RLD}((\operatorname{Ob}H)^{\operatorname{Ob}G};(\operatorname{Ob}H)^{\operatorname{Ob}G})\mid(\operatorname{im}t)\circ G\circ(\operatorname{dom}t)^{-1}\sqsubseteq H\}$.

Conjecture 20. $\operatorname{MOR}_\alpha(G;H)=\operatorname{MOR}_\beta(G;H)$.

Obvious 21.

1. $\operatorname{MOR}_\alpha(G;H)=\bigsqcup\left\{t\in\operatorname{atoms}^{\mathsf{RLD}((\operatorname{Ob}H)^{\operatorname{Ob}G};(\operatorname{Ob}H)^{\operatorname{Ob}G})}\mid\left\langle(\operatorname{dom}t)\times^{(C)}(\operatorname{im}t)\right\rangle G\sqsubseteq H\right\}$;
2. $\operatorname{MOR}_\beta(G;H)=\bigsqcup\left\{t\in\mathsf{RLD}((\operatorname{Ob}H)^{\operatorname{Ob}G};(\operatorname{Ob}H)^{\operatorname{Ob}G})\mid\left\langle(\operatorname{dom}t)\times^{(C)}(\operatorname{im}t)\right\rangle G\sqsubseteq H\right\}$.

Conjecture 22.

1. $t\in\operatorname{atoms}\operatorname{MOR}_a(G;H)\Leftrightarrow(\operatorname{im}t)\circ G\circ(\operatorname{dom}t)^{-1}\sqsubseteq H$;
2. $t\in\operatorname{atoms}\operatorname{MOR}_\beta(G;H)\Leftrightarrow(\operatorname{im}t)\circ G\circ(\operatorname{dom}t)^{-1}\sqsubseteq H$.

Evaluation $\varepsilon\colon(\operatorname{MOR}(G;H)\times G)\to H$ that is for $x\in G=\mathsf{RLD}(\operatorname{Ob}G;\operatorname{Ob}G)$ it is a function on objects defined by the formula:
[TODO: Infer x and y from $x\times y$.]

$$\varepsilon((f\times^{\mathsf{RLD}}g)\times x)=g\circ(x\times^{\mathsf{RLD}}x)\circ f^{-1}=\left\langle f\times^{(C)}g\right\rangle(x\times^{\mathsf{RLD}}x)$$

for atomic $f,g\in\mathfrak{F}((\operatorname{Ob}H)^{\operatorname{Ob}G})$.

$\varepsilon(F\times x)=\bigsqcup\{g\circ(x\times^{\mathsf{RLD}}x)\circ f^{-1}\mid f,g\in\mathfrak{F}((\operatorname{Ob}H)^{\operatorname{Ob}G}),f\times^{\mathsf{RLD}}g\not\asymp F\}$
Let now $f\colon Z\times A\to B$.
In Set $\tilde{f}(a)(b)=f(a;b)$; $\tilde{f}(a)=b\mapsto f(a;b)$
$(\sim f)a=\bigsqcup\{b\times\langle(\mathsf{FCD})f\rangle(a\times b)\mid b\in\operatorname{atoms}^{\mathfrak{F}}\}$ [FIXME: Loss of information, see below.]
Awoday 6.5 "Equational definition" gives a simple way to check cartesian closed categories.
It's enough to prove:

1. $\varepsilon\circ(\sim f\times 1_A)=f$
2. $\varepsilon\circ\sim(g\times 1_A)=g$

Really, $\varepsilon(\sim f\times 1_A)z=\varepsilon(\sim fz\times z)=\varepsilon(\bigsqcup\{b\times\langle(\mathsf{FCD})f\rangle(z\times b)\mid b\in\operatorname{atoms}^{\mathfrak{F}}\}\times z)=\bigsqcup\{g'\circ(z\times^{\mathsf{RLD}}z)\circ f'^{-1}\mid f',g'\in\mathfrak{F}((\operatorname{Ob}H)^{\operatorname{Ob}G}),f'\times^{\mathsf{RLD}}g'\not\asymp\bigsqcup\{b\times\langle(\mathsf{FCD})f\rangle(z\times b)\mid b\in\operatorname{atoms}^{\mathfrak{F}}\}\}$
[FIXME: It cannot be equal to f due loss on information in (FCD).]

5 On decomposisiton of binary relations and reloids

Example 23. $\rho\bigsqcap G\neq\bigsqcap\langle\rho\rangle G$ for some st G of reloids (with matching sources and destinations).

Proof. Take $\Delta=\bigsqcap\{\uparrow^{\mathbb{R}}(-\varepsilon;\varepsilon)\mid\varepsilon>0\}$. Take $\{\alpha\}\times^{\mathsf{RLD}}p$ where $p\sqsubseteq\Delta$ is a nontrivial ultrafilter.

$\langle\rho\bigsqcap G\rangle(\{\alpha\}\times^{\mathsf{RLD}}p)=(\bigsqcap G)\circ(\{\alpha\}\times^{\mathsf{RLD}}p)$

$\langle\bigsqcap\langle\rho\rangle G\rangle(\{\alpha\}\times^{\mathsf{RLD}}p)=$ (because $\{\alpha\}\times^{\mathsf{RLD}}p$ is atomic)$=\bigsqcap\{\langle\rho g\rangle(\{\alpha\}\times^{\mathsf{RLD}}p)\mid g\in G\}=\bigsqcap\{g\circ(\{\alpha\}\times^{\mathsf{RLD}}p)\mid g\in G\}$. □