$f_1 = \bigsqcup \{\mathcal{X} \times^{\mathsf{FCD}} \mathcal{Y} \mid \mathcal{X} \times^{\mathsf{FCD}} \mathcal{Y} \sqsubseteq f_0\}$ what is equal to f_0 because the lattice of funcoids is atomistic and every atom is a funcoidal product.

Let now R_0 be a set of pairs of filters conformung to our axioms, f be a funcoid induced by R by formula (2), R_1 be a set of pairs of filters induced by f by formula (1). We will prove $R_1 = R_0$.

$(\mathcal{X}; \mathcal{Y}) \in R_1 \Leftrightarrow \mathcal{X} \times^{\mathsf{FCD}} \mathcal{Y} \sqsubseteq \bigsqcup \{\mathcal{X} \times^{\mathsf{FCD}} \mathcal{Y} \mid (\mathcal{X}; \mathcal{Y}) \in R_0\} \Leftarrow (\mathcal{X}; \mathcal{Y}) \in R_0$

$\mathcal{X} \times^{\mathsf{FCD}} \mathcal{Y} \sqsubseteq \bigsqcup \{\mathcal{X} \times^{\mathsf{FCD}} \mathcal{Y} \mid (\mathcal{X}; \mathcal{Y}) \in R_0\} \Rightarrow (\mathcal{X}; \mathcal{Y}) \in R_0$ because ??. [FIXME: It seems we need additional axioms!] □

Theorem 4. *If a funcoid is weakly metamonovalued, then it is monovalued.*

Proof. We have $(g \sqcap h) \circ f = (g \circ f) \sqcap (h \circ f)$ for every reloids g, h. We need to prove that f is monovalued.

Prove that exists $F \in \operatorname{up} f$ such that for every g, h we have $(g \sqcap h) \circ F = (g \circ F) \sqcap (h \circ F)$.

it's enough to prove that $(g \sqcap h) \circ F \sqsupseteq (g \circ F) \sqcap (h \circ F)$

Really, ??

thus F is monovalued.

$f \circ f^{-1} = \bigsqcap \{F \circ F^{-1} \mid F \in \operatorname{up} f\}$ □

Proposition 5. *The following statements are equivalent for every endofuncoid μ and a set U:*

1. *U is connected regarding μ.*
2. *For every $a, b \in U$ there exists a totally ordered set $P \subseteq U$ such that $\min P = a$, $\max P = b$, and for every partion $\{X, Y\}$ of P into two sets X, Y such that $\forall x \in X, y \in Y: x < y$, we have $X [\mu]^* Y$.*

Proof.

- **⇐.** Let $A, B \in \mathscr{P}U$ are nonempty. We need to prove $A [\mu]^* B$. We can assume without loss of generality that $A \cap B = \emptyset$. Because A and B are nonempty, we can take $a \in A$ and $b \in B$. ??
- **⇒.** The case $a = b$ is trivial. Assume $a \neq b$. Take $A, B \in \mathscr{P}U$ such that $a \in A$, $b \in B$ and $A \cup B = U$ and $A \cap B = \emptyset$. Then take orders P_A on A with $\min P_A = a$ and P_B on B with $\max P_B = b$. Then the poset $P = P_A + P_B$?? □

1 Directed funcoids

Let $[-\infty; +\infty]$ be the extended real line with the complete funcoid induced by the usual topology on this set.

Proposition 6. *Every ultrafilter on $[-\infty; +\infty]$ converges to exactly one point.*

Proof. It is a well known fact. □

Below is wrong: http://math.stackexchange.com/q/1874451/4876

[FIXME: below is wrong] Use http://math.stackexchange.com/a/1874862/4876

[TODO: $\Delta_+ \to \Delta_>$ or $\Delta_\geq$]

[TODO: Compare without explicit formulas.]

Lemma 7. *For every ultrafilter a and its limit point x:*

0. $\langle [-\infty; +\infty] \rangle a = \Delta(x)$ if $x = a$

1. $\langle [-\infty; +\infty] \rangle a = \Delta_+(x)$ if $x > a$

2. $\langle [-\infty; +\infty] \rangle a = \Delta_-(x)$ if $x < a$

Proof. 0. Obvious.

1. $\langle [-\infty; +\infty] \rangle a = \bigsqcap_{A \in \operatorname{up} a} \langle [-\infty; +\infty] \rangle^* A = \bigsqcap_{A \in \operatorname{up} a, A \sqsubseteq]x; +\infty]} \langle [-\infty; +\infty] \rangle^* A = \bigsqcap_{A \in \operatorname{up} a, A \sqsubseteq]x; +\infty]} \bigsqcup_{y \in A} \Delta(y)$