To prove that it is really a morphism we need to show

$$\left(f_1 \times^{(D)} f_2 \right) \circ Y \sqsubseteq \left(X_1 \times^{(C)} X_2 \right) \circ \left(f_1 \times^{(D)} f_2 \right)$$

that is (for every y)

$$\left(f_1 \times^{(D)} f_2 \right) Y y \sqsubseteq \left(X_1 \times^{(C)} X_2 \right) \left(f_1 \times^{(D)} f_2 \right) y.$$

Really, $\left(f_1 \times^{(D)} f_2 \right) Y y = f_1 Y y \times^{\mathsf{FCD}} f_2 Y y$;

$\left(X_1 \times^{(C)} X_2 \right) \left(f_1 \times^{(D)} f_2 \right) y = \left(X_1 \times^{(C)} X_2 \right) (f_1 y \times^{\mathsf{FCD}} f_2 y) = X_1 f_1 y \times^{\mathsf{FCD}} X_2 f_2 y$;

but it is easy to show $f_1 Y y \times^{\mathsf{FCD}} f_2 Y y \sqsubseteq X_1 f_1 y \times^{\mathsf{FCD}} X_2 f_2 y$.

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I define ??

[TODO: Prove that it is a direct product in **contFcd**.]

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Bibliography

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