Definition 4. I call a **Pos**-morphism *monovalued* when it maps atoms to atoms or least element.

Definition 5. I call a **Pos**-morphism *entirely defined* when its value is non-least on every non-least element.

Obvious 6. A morphism is both monovalued and entirely defined iff it maps atoms into atoms.

[TODO: Show how it relates with dagger categories.]

Definition 7. **mePos** is the subcategory of **Pos** with only monovalued and entirely defined morphisms.

Obvious 8. This is a well defined category.

Definition 9. **mefpFCD** is the subcategory of **fpFCD** with only monovalued and entirely defined morphisms.

Remark 10. In the two above definitions different definitions of monovaluedness and entire definedness from different articles.

4 Definition of the categories

Definition 11. A *(pointfree) endo-funcoid* is a (pointfree) funcoid with the same source and destination (an endomorphism of the category of (pointfree) funcoids). I will denote $\operatorname{Ob} f$ the object of an endomorphism f.

Obvious 12. The *category of continuous pointfree funcoids* **cont**(**fpFCD**) is:

- Objects are small pointfree endo-funcoids.
- Morphisms from an object a to an object b are triples $(f; a; b)$ where f is a pointfree funcoid from $\operatorname{Ob} a$ to $\operatorname{Ob} b$ such that f is a continuous morphism from a to b (that is $f \circ a \sqsubseteq b \circ f$, or equivalently $a \sqsubseteq f^{-1} \circ b \circ f$, or equivalently $f \circ a \circ f^{-1} \sqsubseteq f$).
- Composition is the composition of pointfree funcoids.
- Identity for an object a is $(I^{\mathsf{FCD}}_{\operatorname{Ob} a}; a; a)$.

5 Isomorphisms

Theorem 13. If f is an isomorphism $a \rightarrow b$ of the category **cont**(**fpFCD**), then:

1. $f \circ a = b \circ f$;
2. $a = f^{-1} \circ b \circ f$;
3. $f \circ a \circ f^{-1} = b$.

Proof. Note that f is monovalued and entirely defined.

1. We have $f \circ a \sqsubseteq b \circ f$ and $f^{-1} \circ b \sqsubseteq a \circ f^{-1}$. Consequently $f^{-1} \circ f \circ a \sqsubseteq f^{-1} \circ b \circ f$; $a \sqsubseteq f^{-1} \circ b \circ f$; $a \circ f^{-1} \sqsubseteq f^{-1} \circ b \circ f \circ f^{-1}$; $a \circ f^{-1} \sqsubseteq f^{-1} \circ b$. Similarly $b \circ f \sqsubseteq f \circ a$. So $f \circ a = b \circ f$.

2 and 3. Follow from the definition of isomorphism. □

Isomorphisms are meant to preserve structure of objects. I will show that (under certain conditions) isomorphisms of **cont**(**fpFCD**) really preserve structure of objects.

First we will consider an isomorphism between objects a and b which are funcoids (not the general case of pointfree funcoids). In this case a map which preserves structure of objects is a *bijection*. It is really a bijection as the following theorem says: