Categories related with funcoids

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Abstract

I consider some categories related with pointfree funcoids.

1 Draft status

This is a rough partial draft.

2 Topic of this article

In this article are considered some categories related to *pointfree funcoids* [1].

3 Category of continuous morphisms

I will denote $\operatorname{Ob} f$ the object (source and destination) of an endomorphism f.

Definition 1. Let C is a partially ordered category. The category $\mathbf{cont}(C)$ (which I call *the category of continuous morphism* over C) is:

- Objects are endomorphisms of category C.
- Morphisms are triples $(f; a; b)$ where a and b are objects and $f: \operatorname{Ob} a \to \operatorname{Ob} b$ is a morphism of the category C such that $f \circ a \sqsubseteq b \circ f$.
- Composition of morphisms is defined by the formula $(g; b; c) \circ (f; a; b) = (g \circ f; a; c)$.
- Identity morphisms are $(a; a; 1_a^C)$.

It is really a category:

Proof. We need to prove that: composition of morphisms is a morphism, composition is associative, and identity morphisms can be canceled on the left and on the right.

That composition of morphisms is a morphism follows from these implications:

$$f \circ a \sqsubseteq b \circ f \wedge g \circ b \sqsubseteq c \circ g \Rightarrow g \circ f \circ a \sqsubseteq g \circ b \circ f \sqsubseteq c \circ g \circ f.$$

That composition is associative is obvious.

That identity morphisms can be canceled on the left and on the right is obvious. □

Remark 2. The "physical" meaning of this category is:

- Objects (endomorphisms of C) are spaces.
- Morphisms are continuous functions between spaces.
- $f \circ a \sqsubseteq b \circ f$ intuitively means that f combined with an infinitely small is less than infinitely small combined with f (that is f is continuous).

Remark 3. Every $\operatorname{Hom}(\mathfrak{A}; \mathfrak{B})$ of **Pos** is partially ordered by the formula $a \leqslant b \Leftrightarrow \forall x \in \mathfrak{A}: a(x) \leqslant b(x)$. So $\mathbf{cont}(\mathbf{Pos})$ is defined.