



FIGURE 1. Examples of surface cardinality

**FixMe:** Try to replace isomorphism  $f$  with some kind of filter embedding.

Consider the dihedral angle  $T$  produced by two half-planes. Are the points of intersection of the half-planes isomorphism-special? (They should not be considered special. If they are special, this is a probably flaw in the definition of isomorphism special.)

Consider union  $T$  of two intersecting lines on a plane. The intersection may be considered as a special point, because it has more connected components than the rest. We don't want to consider it special, however. We can restrict to consider special only points which have less connected components (rather than more) to correct this trouble. Also try to define it with some kind of morphisms of filters instead of isomorphism as in isomorphism-special.

EXERCISE 2433. Excluding special points (either cardinality or isomorphism) from closed disk produces open disk.

Let us note that special points of closed disk have surface cardinality 1 which is less than surface cardinality (2) of regular points. So, it is a conceivable idea to consider special points which have lesser surface cardinality than nearby points.

Consider the following two subsets of a plane (the lines are the set  $T$ , the small black blob is the point  $a$ , and the cyan blob symbolizes the filter  $\langle \mu \rangle^* \{a\} \setminus T$ ):

For one of the sets surface cardinality of  $a$  is 3 and for another it is 2.

Now define *shift special points*.

Let  $I$  be an interval on  $\mathbb{R}$  (containing zero?)

A point  $a$  is *shift special* if there exists a transformation (that is a continuous function  $f : I \times \mu \rightarrow \mu$  such that:

- 1°.  $f(0)$  is identity. **FixMe:** Is this condition needed?
- 2°. for every sufficiently small  $\epsilon > 0$  we have  $f(\epsilon, a) \in T$ ;
- 3°. there is  $\epsilon > 0$  such that for every  $0 < \epsilon' < \epsilon$  we have  $f(\epsilon')$  being not continuous at  $a$  regarding complete funcooid defined by the function  $x \mapsto \langle \mu \rangle^* \{x\} \setminus T$ .

We may consider to additionally require that every  $f(\epsilon)$  is isomorphism of funcooids.

EXAMPLE 2434.  $T$  is disk  $\left\{ \frac{(x,y,0)}{x^2+y^2 \leq 1} \right\}$ .  $f$  is the contraction  $(\epsilon, v) \mapsto \frac{1}{1+\epsilon}v$ .  $a = (1, 0, 0)$ .

In the usual topology  $f$  is continuous. In  $x \mapsto \langle \mu \rangle^* \{x\} \setminus T$  we have the function  $\epsilon \mapsto f(\epsilon)$  not continuous at zero. So  $a$  is a shift special point.

PROOF.  $f(0)(v) = v$ . Thus  $\langle f(0) \rangle (\langle \mu \rangle^* \{a\} \setminus T) = \langle \mu \rangle^* \{a\} \setminus T$  intersects the plane  $Z = 0$ . But  $f(0, a)$

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QUESTION 2435. Can we exclude real numbers from the play?

QUESTION 2436. How cardinality special points, isomorphism special points and shift special points are related with each others?