CHAPTER 16

Manifolds and surfaces

1. Sides of a surface

DEFINITION 2427. Let μ be an endofuncoid on a set U. *Surface side* of a set $T \subseteq \operatorname{Ob}\mu$ is a connected component (regarding μ) of the filter $(\langle\mu\rangle^* T) \setminus T$. FiXme: μ is used twice in this definition. We may generalize for two different funcoids instead.

Keep in mind that the above definition may work nicely if μ is a complete funcoid induced by a topological space.

EXAMPLE 2428. For an $\mathbb{R}^{n-1}$ subspace T of a $\mathbb{R}^n$ ($n \geq 1$) euclidean space and the complete funcoid μ induced by the usual topology:

1°. T has exactly two surface sides.
2°. The filter $\langle\mu\rangle^* @\{a\} \setminus T$ (for every $a \in T$) has exactly two connected components.

PROOF. Without loss of generality assume that

$$T = \left\{ \frac{(x_0, x_1, \dots, x_{n-2}, 0)}{x_0, x_1, \dots, x_{n-2} \in \mathbb{R}} \right\}; \quad a = (0, \dots, 0).$$

We have

$$\langle\mu\rangle^* @\{a\} = \left(\uparrow \left\{ \frac{v \in \mathbb{R}^n}{v_{n-1} > 0} \right\} \sqcap \langle\mu\rangle^* @\{a\} \right) \sqcup \left(\uparrow \left\{ \frac{v \in \mathbb{R}^n}{v_{n-1} < 0} \right\} \sqcap \langle\mu\rangle^* @\{a\} \right).$$

Let us prove that $\uparrow \left\{ \frac{v \in \mathbb{R}^n}{v_{n-1} > 0} \right\} \sqcap \langle\mu\rangle^* @\{a\}$ and $\uparrow \left\{ \frac{v \in \mathbb{R}^n}{v_{n-1} < 0} \right\} \sqcap \langle\mu\rangle^* @\{a\}$ are connected components.

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1.1. Special points.

We will start from the example of open $T = \left\{ \frac{(x,y,0)}{x^2+y^2<1} \right\}$ and closed $T = \left\{ \frac{(x,y,0)}{x^2+y^2 \leq 1} \right\}$ disks in $\mathbb{R}^3$.

EXERCISE 2429. Prove that open disk (in a usual 3-dimensional space) has two surface sides and closed disk has one surface side.

2. Special points

DEFINITION 2430. *Surface cardinality* of a point a (an element of the set $\operatorname{Ob}\mu$) is the cardinality of the set of connected components of the filter $\langle\mu\rangle^*\{a\} \setminus T$.

DEFINITION 2431. *Cardinality regular point* is a point a, which has a neighborhood ($X \in \operatorname{up}\langle\mu\rangle^*\{a\}$) such that all points $x \in X \cap T$ are of the same surface cardinality as the point a.

Cardinality special point is a point which is not cardinality regular.

DEFINITION 2432. *Isomorphism regular point* is a point a, which has a neighborhood ($X \in \operatorname{up}\langle\mu\rangle^*\{a\}$) such that for all points $x \in X \cap T$ the filter $\langle\mu\rangle^*\{a\}$ is isomorphic to $\langle\mu\rangle^*\{x\}$.

Isomorphism special point is a point which is not isomorphism regular.