CHAPTER 15

Relationships are pointfree funcoids

THEOREM 2422. $((\mathsf{FCD}), (\mathsf{RLD})_{\mathrm{in}})$ are components of a complete pointfree funcoid.

PROOF. For every ultrafilters x and y we have $x\ [(\mathsf{FCD})(f \sqcap (\mathsf{RLD})_{\mathrm{in}} g)]\ y \Leftrightarrow x \times^{\mathsf{RLD}} y \not\asymp f \sqcap (\mathsf{RLD})_{\mathrm{in}} g \Leftrightarrow x \times^{\mathsf{RLD}} y \sqsubseteq (\mathsf{RLD})_{\mathrm{in}} g \wedge x \times^{\mathsf{RLD}} y \not\asymp f \sqcap (\mathsf{RLD})_{\mathrm{in}} g \Leftrightarrow x \times^{\mathsf{FCD}} y \in \operatorname{atoms} g : x \times^{\mathsf{RLD}} y \not\asymp f \sqcap (\mathsf{RLD})_{\mathrm{in}} g \Leftrightarrow x \times^{\mathsf{FCD}} y \in \operatorname{atoms} g : x \times^{\mathsf{RLD}} y \not\asymp f \Leftrightarrow x \times^{\mathsf{FCD}} y \in \operatorname{atoms} g \wedge x \times^{\mathsf{FCD}} y \sqsubseteq (\mathsf{FCD}) f \Leftrightarrow x\ [g \sqcap (\mathsf{FCD}) f]\ y$.

Thus $(\mathsf{FCD})(f \sqcap (\mathsf{RLD})_{\mathrm{in}} g) = g \sqcap (\mathsf{FCD}) f$. Consequently $f \sqcap (\mathsf{RLD})_{\mathrm{in}} g = \bot \Leftrightarrow g \sqcap (\mathsf{FCD}) f = \bot$ that is $g \not\asymp (\mathsf{FCD}) f \Leftrightarrow f \not\asymp (\mathsf{RLD})_{\mathrm{in}} g$.

It is complete by theorem 1198. □

We will also prove in another way that (FCD), $(\mathsf{RLD})_{\mathrm{in}}$ are components of pointfree funcoids:

THEOREM 2423. $(\mathsf{RLD})_{\mathrm{in}}$ is a component of a pointfree funcoid (between filters on boolean lattices).

PROOF. Consider the pointfree funcoid $\mathscr{R}$ defined by the formula $\langle \mathscr{R} \rangle^* F = (\mathsf{RLD})_{\mathrm{in}} F$ for binary relations F (obviously it does exists). Then $\langle \mathscr{R} \rangle f = \langle \mathscr{R} \rangle \bigsqcap^{\mathsf{FCD}} \mathrm{up}^{\Gamma} f = \bigsqcap^{\mathsf{RLD}}_{F \in \mathrm{up}^{\Gamma} f} \langle \mathscr{R} \rangle^* F = \bigsqcap^{\mathsf{RLD}}_{F \in \mathrm{up}^{\Gamma} f} (\mathsf{RLD})_{\mathrm{in}} F = (\mathsf{RLD})_{\mathrm{in}} \bigsqcap^{\mathsf{FCD}}_{F \in \mathrm{up}^{\Gamma} f} F = (\mathsf{RLD})_{\mathrm{in}} f$. □

THEOREM 2424. (FCD) is a component of a complete pointfree funcoid (between filters on boolean lattices).

PROOF. Consider the pointfree funcoid $\mathscr{Q}$ defined by the formula $\langle \mathscr{Q} \rangle^* F = (\mathsf{FCD}) F$ for binary relations F (obviously it does exists). Then $\langle \mathscr{Q} \rangle f = \langle \mathscr{Q} \rangle \bigsqcap^{\mathsf{RLD}} \mathrm{up}\, f =$ (because $\mathrm{up}\, f$ is a filter base) $= \bigsqcap^{\mathsf{FCD}}_{F \in \mathrm{up}\, f} \langle \mathscr{Q} \rangle^* F = \bigsqcap^{\mathsf{FCD}}_{F \in \mathrm{up}\, f} (\mathsf{FCD}) F = \bigsqcap^{\mathsf{FCD}}_{F \in \mathrm{up}\, f} F = \bigsqcap^{\mathsf{FCD}} \mathrm{up}\, f = (\mathsf{FCD}) f$. □

PROPOSITION 2425. $(\mathsf{FCD}) \bigsqcap S = \bigsqcap_{f \in S} (\mathsf{FCD}) f$ if S is a filter base of reloids (with the same sources and destinations).

PROOF. Theorem 831. □

CONJECTURE 2426. $(\mathsf{RLD})_{\mathrm{in}} \bigsqcap S = \bigsqcap_{f \in S} (\mathsf{RLD})_{\mathrm{in}} f$ if S is a filter base of funcoids (with the same sources and destinations).