Also for S^*

EXAMPLE 2402. Connected components may not form a weak partition.

PROOF. Consider funcoid $1^{\mathsf{FCD}(\mathbb{R})} \sqcup (\Delta \times^{\mathsf{FCD}} \Delta)$ on real line. Then connected components are (prove!) non-zero singletons and Δ. It is not a weak partition. □

CONJECTURE 2403. If the set of connected components is finite, then it is a strong partition. Moreover the set of connected components is a tearing.

Add more counter-examples (for non-principal filters).

OBVIOUS 2404. Improper filter $\bot^{\mathscr{F}}$ is connected regarding:

1°. every funcoid;
2°. every reloid.

PROPOSITION 2405. The only filter connected regarding

1°. $\bot^{\mathsf{FCD}(A)}$;
2°. $\bot^{\mathsf{RLD}(A)}$

is the improper filter $\bot^{\mathscr{F}}$.

PROOF.

1°. Let $\mathcal{A}$ be a filter. Take $\mathcal{X} = \mathcal{Y} = \mathcal{A} \in \mathscr{F}(\operatorname{Ob}\mu) \setminus \{\bot\}$. Then $\mathcal{X} \sqcup \mathcal{Y} = \mathcal{A}$ but not $\mathcal{X}\ [\mu]\ \mathcal{Y}$.
2°. $S_1^*(\bot^{\mathsf{RLD}(A)}) = S_1(\bot^{\mathsf{RLD}(A)}) = \bot^{\mathsf{RLD}(A)}$. Thus the only connected filter is $\bot^{\mathscr{F}}$.

□

PROPOSITION 2406. Connected filters regarding

1°. $1^{\mathsf{FCD}(A)}$;
2°. $1^{\mathsf{RLD}(A)}$

are exactly ultrafilters and the improper filter.

PROOF. 1. That ultrafilters are connected follows from the fact that for every non-least $\mathcal{X}$, $\mathcal{Y}$ such that $\mathcal{X} \sqcup \mathcal{Y} = \mathcal{A}$ we have $\mathcal{X} = \mathcal{Y} = \mathcal{A}$ and thus $\mathcal{X}\ [1^{\mathsf{FCD}(A)}]\ \mathcal{Y}$. So ultrafilters are connected; so is improper filter too, because improper filter is always connected.

It remains to prove that filters containing more than one distinct ultrafilter are not connected. Really let distinct ultrafilters $a, b \in \operatorname{atoms}\mathcal{A}$. Then not $a\ [1^{\mathsf{FCD}(A)}]\ b$. Thus $\mathcal{A}$ is not connected.

2. A filter a is connected iff $S_1^*(1^{\mathsf{RLD}(A)} \sqcap (a \times^{\mathsf{RLD}} a)) \sqsupseteq a \times^{\mathsf{RLD}} a$ that is iff $S_1^*(\mathrm{id}_a^{\mathsf{RLD}}) \sqsupseteq a \times^{\mathsf{RLD}} a$,

$\bigsqcap_{F \in \operatorname{up}\mathrm{id}_a^{\mathsf{RLD}}} S_1(F) \sqsupseteq a \times^{\mathsf{RLD}} a$ what by properties of generalized filter bases is equivalent to $\bigsqcap_{A \in \operatorname{up} a} S_1(\mathrm{id}_A) \sqsupseteq a \times^{\mathsf{RLD}} a$; $\bigsqcap_{A \in \operatorname{up} a} \mathrm{id}_A \sqsupseteq a \times^{\mathsf{RLD}} a$; $\mathrm{id}_a^{\mathsf{RLD}} \sqsupseteq a \times^{\mathsf{RLD}} a$. This is true exactly for ultrafilters and the improper filter. □

DEFINITION 2407. A *path* regarding funcoid μ is a tuple $p_0, \ldots, p_n$ ($n \in \mathbb{N}$) of atomic filters such that $p_i\ [\mu]\ p_{i+1}$ for every $i = 0, \ldots, n-1$.

The number n is called *path length*.

A path is *between* atomic filters a and b iff $p_0 = a$ and $p_n = b$.

EXAMPLE 2408. $\mu \sqsupseteq \mathrm{id}_{\mathcal{A}}^{\mathsf{FCD}}$ is not necessary for a filter $\mathcal{A}$ to be connected regarding a funcoid μ. Moreover $\mu \sqsupseteq 1^{\mathsf{FCD}}$ is not necessary for a filter $\top$ to be connected regarding a funcoid μ.