Prove $\left\langle \nu^{-1} \right\rangle^* \langle U^q \rangle^* A \sqsubseteq \left\langle \nu^{-1\circ} \right\rangle^* \left\langle \nu^{-1} \right\rangle^* \langle U^r \rangle^* A$ for any $q < r$ in $\mathbb{Q}_2$. FiXme: Can be easily rewritten with the formula $\langle \nu \rangle^* \left\langle \nu^{-1} \right\rangle^* \langle U^q \rangle^* A \sqsubseteq \left\langle \nu^{-1} \right\rangle^* \langle U^r \rangle^* A$ instead. It may extend to non-complete funcoids.

There is such l that $0 = q_l < r_l = 1$ and $q_i = r_i$ for all $i < l$.

It follows $l_q \neq l \leq l_r$.

Consider variants:

$l_q < l$. $\left\langle \nu^{-1} \right\rangle^* \langle U^q \rangle^* A \sqsubseteq \left\langle \nu^{-1} \right\rangle^* \left\langle U_{l_q} \circ \ldots \circ U_1^{q_1 q_{l_q}} \right\rangle^* A =$
$\left\langle \nu^{-1} \right\rangle^* \left\langle U_{l_q}^{r_{l_q}} \circ \ldots \circ U_1^{r_1} \right\rangle^* A \sqsubseteq \langle \nu^{-1} \rangle^* \langle U_{l-1}^{r_{l-1}} \circ \ldots \circ U_1^{r_1} \rangle^* A \sqsubseteq$
$\langle \nu^{-1\circ} \rangle^* \left\langle \nu^{-1} \right\rangle^* \langle U_l^{r_l} \circ U_{l-1}^{r_{l-1}} \circ \ldots \circ U_1^{r_1} \rangle^* A = \langle \nu^{-1\circ} \rangle^* \left\langle \nu^{-1} \right\rangle^* \langle U^r \rangle^* A$
(use $U_l^{r_l} \in \mathrm{up}(\mathsf{FCD})\mu$ by theorem 992).

$l < l_q$. Inclusions $U_k \circ U_k \sqsubseteq U_{k-1}$ for $l < k \leq l_q + 1$ guarantee that $U_{l_q+1} \circ U_{l_q} \circ \ldots \circ U_{l+1} \sqsubseteq U_l$ and then $\left\langle \nu^{-1} \right\rangle^* \langle U^q \rangle^* A \sqsubseteq \left\langle \nu^{-1} \right\rangle^* \left\langle U_{l_q}^{q_{l_q}} \circ \ldots \circ U_1^{q_1} \right\rangle^* A \sqsubseteq$
$\left\langle \nu^{-1\circ} \right\rangle^* \left\langle \nu^{-1} \right\rangle^* \left\langle U_{l_q+1}^{q_{l_q+1}} \circ U_{l_q}^{q_{l_q}} \circ \ldots \circ U_1^{q_1} \right\rangle^* A =$
$\left\langle \nu^{-1\circ} \right\rangle^* \left\langle \nu^{-1} \right\rangle^* \left\langle U_{l_q+1} \circ U_{l_q}^{q_{l_q}} \circ \ldots \circ U_l^0 \circ \ldots \circ U_1^{q_1} \right\rangle^* A \sqsubseteq$
$\left\langle \nu^{-1\circ} \right\rangle^* \left\langle \nu^{-1} \right\rangle^* \langle U_l \circ U_{l-1}^{q_{l-1}} \circ \ldots \circ U_1^{q_1} \rangle^* A \sqsubseteq$
$\left\langle \nu^{-1\circ} \right\rangle^* \langle \nu^{-1} \rangle^* \left\langle U_l^{r_l} \circ U_{l-1}^{r_{l-1}} \circ \ldots \circ U_1^{r_1} \right\rangle^* A \sqsubseteq$
$\langle \nu^{-1\circ} \rangle^* \left\langle \nu^{-1} \right\rangle^* \left\langle U_{l_r}^{r_{l_r}} \circ \ldots \circ U_1^{r_1} \right\rangle^* A = \left\langle \nu^{-1\circ} \right\rangle^* \left\langle \nu^{-1} \right\rangle^* \langle U_r \rangle^* A.$

Define f by the formula $f(z) = \inf\left(\{1\} \cup \left\{ \frac{q \in \mathbb{Q}_2}{z \in \langle \nu^{-1} \rangle^* \langle U^q \rangle^* A} \right\} \right)$.

It is clear?? that $A \sqsubseteq \left\langle f^{-1} \right\rangle^* \{0\}$ and $\left\langle f^{-1} \right\rangle^* [0;1[\sqsubseteq \bigcup_{q \in \mathbb{Q}_2} \left\langle \nu^{-1} \right\rangle^* \langle U^q \rangle^* A = \bigcup_{r \in \mathbb{Q}_2} \langle \nu^{-1\circ} \rangle^* \left\langle \nu^{-1} \right\rangle^* \langle U^r \rangle^* A \sqsubseteq \left\langle \nu^{-1\circ} \right\rangle^* \langle \nu^{-1} \rangle^* \langle U_0 \rangle^* A$.

To prove that the map $f : X \to [0,1]$ is continuous, it suffices to check that for every real number $a \in]0;1[$ the sets $\langle f^{-1} \rangle^* [0;a[$ and $\left\langle f^{-1} \right\rangle^*]a;1]$ are open. This follows from the equalitites

$\left\langle f^{-1} \right\rangle^* [0;a[= \bigcup_{\mathbb{Q}_2 \ni q < a} \left\langle \nu^{-1\circ} \right\rangle^* \left\langle \nu^{-1} \right\rangle^* \langle U^q \rangle^* A$ and $\left\langle f^{-1} \right\rangle^*]a;1] = \bigcup_{\mathbb{Q}_2 \ni r > a} (X \setminus \langle \nu^{-1} \rangle^* \langle U^r \rangle^* A)$. □

How the formulas for normal (T_4) topological spaces and normal quasi-uniformities are related? Maybe this works: Replacing $\nu \to \mu \circ \mu^{-1}$, $\mu \to 1$ makes $\nu \circ \nu^{-1} \sqsubseteq \nu^{-1} \circ (\mathsf{FCD})\mu \to \mu \circ \mu^{-1} \circ \mu \circ \mu^{-1} \sqsubseteq \mu \circ \mu^{-1}$.

https://www.researchgate.net/project/The-lattice-LG-of-group-topologies