Because D was taken arbitrary, we have $a \times^{\mathsf{RLD}} a \sqsubseteq \nu$ that is a is Cauchy.

$\Leftarrow$. Suppose that Cauchy space associated with a reloid ν is totally bounded but the reloid ν isn't totally bounded. So there exists a $D \in \operatorname{GR} \nu$ such that $(\operatorname{Ob} \nu) \setminus \langle D \rangle F \neq \emptyset$ for every finite set F.

Consider the filter base

$$S = \left\{ \frac{(\operatorname{Ob} \nu) \setminus \langle D \rangle F}{F \in \mathscr{P} \operatorname{Ob} \nu, F \text{ is finite}} \right\}$$

and the filter $\mathcal{F} = \bigsqcap \langle \uparrow^{\operatorname{Ob} \nu} \rangle S$ generated by this base. The filter $\mathcal{F}$ is proper because intersection $P \cap Q \in S$ for every $P, Q \in S$ and $\emptyset \notin S$. Thus there exists a Cauchy (for our Cauchy space) filter $\mathcal{X} \sqsubseteq \mathcal{F}$ that is $\mathcal{X} \times^{\mathsf{RLD}} \mathcal{X} \sqsubseteq \nu$.

Thus there exists $M \in \mathcal{X}$ such that $M \times M \subseteq D$. Let F be a finite subset of $\operatorname{Ob} \nu$. Then $(\operatorname{Ob} \nu) \setminus \langle D \rangle F \in \mathcal{F} \sqsupseteq \mathcal{X}$. Thus $M \not\asymp (\operatorname{Ob} \nu) \setminus \langle D \rangle F$ and so there exists a point $x \in M \cap ((\operatorname{Ob} \nu) \setminus \langle D \rangle F)$.

$\langle M \times M \rangle \{p\} \subseteq \langle D \rangle \{x\}$ for every $p \in M$; thus $M \subseteq \langle D \rangle \{x\}$.

So $M \subseteq \langle D \rangle (F \cup \{x\})$. But this means that $M \in \mathcal{X}$ does not intersect $(\operatorname{Ob} \nu) \setminus \langle D \rangle (F \cup \{x\}) \in \mathcal{F} \sqsupseteq \mathcal{X}$, what is a contradiction (taken into account that $\mathcal{X}$ is proper).

□

http://math.stackexchange.com/questions/104696/pre-compactness-total-boundedness-and-cauchy-sequential-compactness

13. Totally bounded funcoids

DEFINITION 2370. A funcoid ν is totally bounded iff

$$\forall X \in \operatorname{Ob} \nu \exists \mathcal{X} \in \mathfrak{F}^{\operatorname{Ob} \nu} : (0 \neq \mathcal{X} \sqsubseteq \uparrow^{\operatorname{Ob} \nu} X \wedge \mathcal{X} \times^{\mathsf{FCD}} \mathcal{X} \sqsubseteq \nu).$$

This can be rewritten in elementary terms (without using funcoidal product: $\mathcal{X} \times^{\mathsf{FCD}} \mathcal{X} \sqsubseteq \nu \Leftrightarrow \forall P \in \partial \mathcal{X} : \mathcal{X} \sqsubseteq \langle \nu \rangle P \Leftrightarrow \forall P \in \partial \mathcal{X}, Q \in \partial \mathcal{X} : P \; [\nu]^* \; Q \Leftrightarrow \forall P, Q \in \operatorname{Ob} \nu : (\forall E \in \mathcal{X} : (E \cap P \neq \emptyset \wedge E \cap Q \neq \emptyset) \Rightarrow P \; [\nu]^* \; Q)$.

Note that probably I am the first person which has written the above formula (for proximity spaces for instance) explicitly.

14. On principal low spaces

DEFINITION 2371. A low space $(U, \mathscr{C})$ is *principal* when all filters in $\mathscr{C}$ are principal.

PROPOSITION 2372. Having fixed a set U, principal reflexive low spaces on U bijectively correspond to principal reflexive symmetric endoreloids on U.

PROOF. ??

http://math.stackexchange.com/questions/701684/union-of-cartesian-squares □

15. Rest

https://en.wikipedia.org/wiki/Cauchy_filter#Cauchy_filters

https://en.wikipedia.org/wiki/Uniform_space "Hausdorff completion of a uniform space" here)

http://at.yorku.ca/z/a/a/b/13.htm : the category **Prox** of proximity spaces and proximally continuous maps (i.e. maps preserving nearness between two sets) is isomorphic to the category of totally bounded uniform spaces (and uniformly continuous maps).